ASTR367

HW#3

1) $$\frac{dM}{dr} = 4\pi r^2 \rho \qquad \frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$$

$$\Rightarrow \frac{dP}{dM} = -\frac{GM_r \cancel{\rho}}{r^2} \cdot \frac{1}{4\pi r^2 \cancel{\rho}}$$

$$= -\frac{GM_r}{4\pi r^4}$$

$$dP = -\frac{GM_r \, dM}{r^4}$$

$$\int_{P_c}^{0} dP = -G \int_{0,0}^{M_\odot, R_\odot} \frac{M_r}{r^4} \, dM = \frac{G}{2R_\odot^4} M_\odot^2 = P_c$$

$$P_c = \frac{6.67\times10^{-11}\ N\ m^2/kg^2}{2\times(7\times10^8\ m)^4} (2\times10^{30}\ kg)^2 = 5.56\times10^{14}\ N\ m^{-2}$$

2) The Eddington luminosity is the maximum luminosity a star can have before radiation outward exerts more force than gravity inward.

$$F_g = \frac{G M_* m}{r^2} \quad \text{test mass}$$

$$F_{rad} = P_{rad}\,\kappa m = \frac{L}{c}\frac{\kappa m}{4\pi r^2}$$

$$F_g = F_{rad} \Rightarrow \frac{G M_* \cancel{m}}{\cancel{r^2}} = \frac{L}{c}\frac{\kappa \cancel{m}}{4\pi \cancel{r^2}}$$

$$L = \frac{4\pi G c}{\kappa} M = \frac{4\pi G c}{\sigma_T / m_p} M$$

$$\frac{L}{L_\odot} = \frac{4\pi G c}{\sigma_T / m_p} \frac{M_\odot}{L_\odot} \cdot \frac{M}{M_\odot}$$

b) $$\left(\frac{M}{M_\odot}\right)^{3.5} = \frac{4\pi G c}{\sigma_T / m_p} \frac{M_\odot}{L_\odot} \frac{M}{M_\odot}$$

$$\Rightarrow \frac{M}{M_\odot} = \left(\frac{4\pi G c}{\sigma_T / m_p} \frac{M_\odot}{L_\odot}\right)^{1/2.5} = 64$$

3) $\epsilon_{pp} \cong \epsilon'_{0,pp}\, \rho\, X^2 f_{pp} \psi_{pp} C_{pp} T_6^4$

$\epsilon_{CNO} \cong \epsilon'_{0,pp}\, \rho\, X\, X_{CNO}\, T_6^{19.9}$

$\epsilon_{pp} = \epsilon_{CNO}$

$\Rightarrow \epsilon'_{0,pp}\, \rho\, X^2\, T_6^4 = \epsilon'_{0,CNO}\, \rho\, X\, X_{CNO}\, T_6^{19.9}$

$$T_6^{15.9} = \frac{\epsilon'_{0,pp}}{\epsilon'_{0,CNO}} \frac{X}{X_{CNO}}$$

$$T_6 = \left(\frac{\epsilon'_{0,pp}}{\epsilon'_{0,CNO}} \frac{X}{X_{CNO}} \right)^{1/15.9}$$

Assume $X = 0.34$, $X_{CNO} = 0.013$, values for $\epsilon_{,0}$ from book

$T_6 = 16.9$ or $T = 1.69 \times 10^7$ K

b) Just plug + chug. <u>but</u>

$\rho = 162\ g\ cm^{-3} = 1.62 \times 10^5\ kg\ m^{-3}$

plugging in, we get

$\epsilon_{pp} = 0.0013$ W/kg

$\epsilon_{CNO} = 0.00042$ W/kg

4) In every reaction the sun loses 0.7% of the mass of 4 protons.

Using $E = mc^2$,

$$\frac{dE}{dt} = L = \frac{dm}{dt} c^2$$

$$\frac{dm}{dt} = \frac{L}{c^2} = 4.26 \times 10^9 \text{ kg/s} = 6.69 \times 10^{-14} \, M_\odot/\text{yr}$$

b) $$\tau = \frac{0.1 \times 0.007}{6.69 \times 10^{-14}} = 1.04 \times 10^{10} \text{ yr}$$

c) $$\frac{L}{L_\odot} = 85^{3.5} = 5.66 \times 10^6$$

Since $\tau \propto \frac{M}{L}$, $$\frac{\tau}{\tau_\odot} = \frac{85}{5.66 \times 10^6} = 1.50 \times 10^{-5}$$

or $\tau = 1.58 \times 10^5$ yr

d) $$\frac{\tau}{\tau_\odot} = \frac{0.08}{0.08^{3.5}} = 552$$ or $\tau = 5.74 \times 10^{12}$ yr