ASTR 702
Interpreting Blackbody Emission

We generally assume that stars emit as blackbodies, which is actually not a terrible assumption for most purposes. Let's explore the basic properties of blackbodies so we can apply this knowledge to stars.

The blackbody (Planck function) is:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\,. \tag{1}$$

or

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}\,, \tag{2}$$

where the function is evaluated at frequency ν or wavelength λ, and the object is at temperature T. These functions are shown in Figure 2 for different temperatures.

Students are often confused by the units: $\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}\,sr^{-1}}$ ($\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$) for B_ν or $\mathrm{erg\,cm^{-2}\,s^{-1}\,cm^{-1}\,sr^{-1}}$ ($\mathrm{W\,m^{-2}\,cm^{-1}\,sr^{-1}}$)for B_λ, where the additional "Hz" or "cm" term is the frequency or wavelength (often given in Angstroms). This also means that it is a surface brightness or an intensity.

As we learned last lecture, the fundamental observational quantity in astronomy is the specific intensity I_ν. But under what conditions is $I_\nu = B_\nu$? When something called the

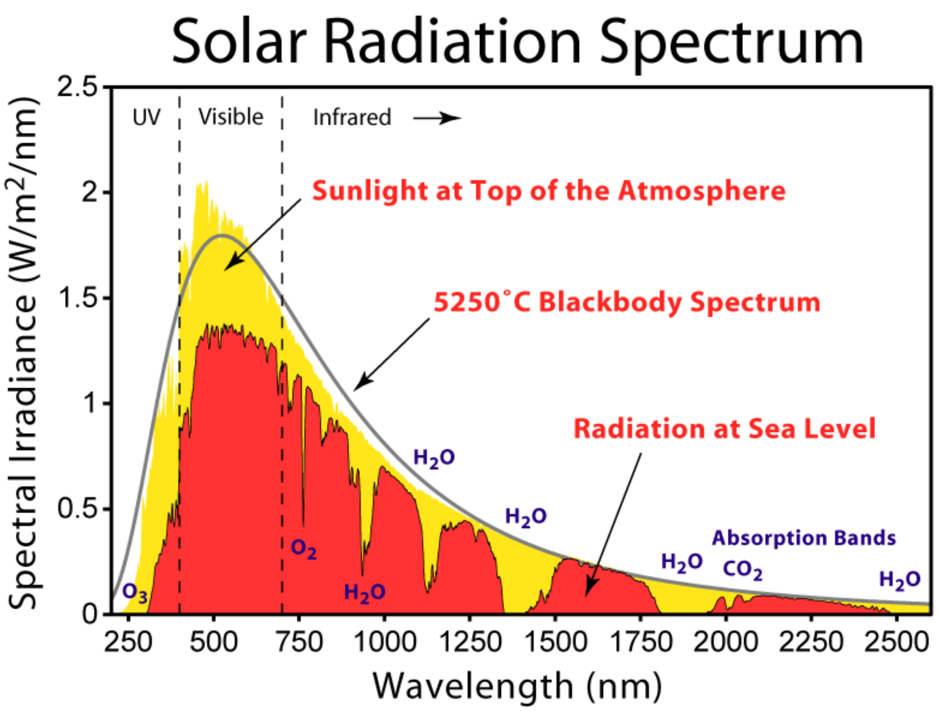


Figure 1: Comparison of Solar and blackbody spectra.

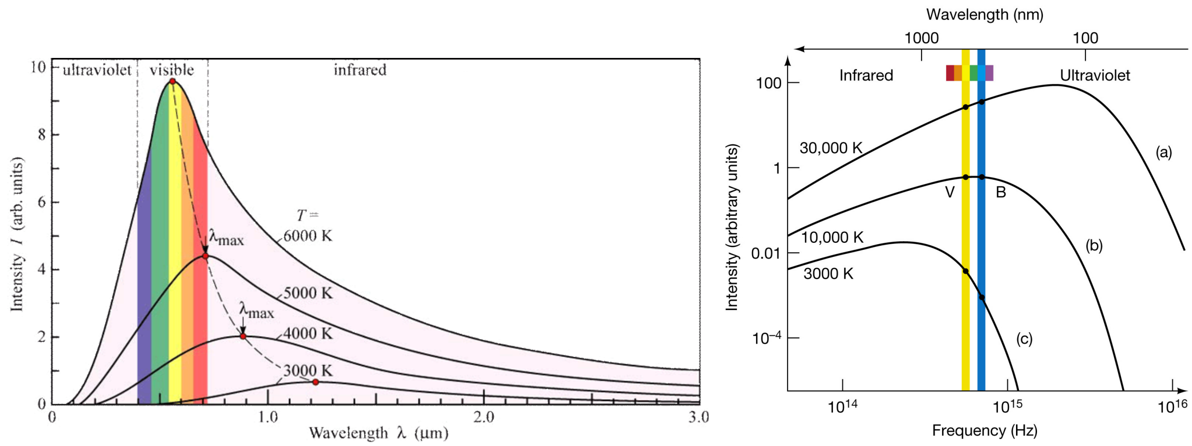
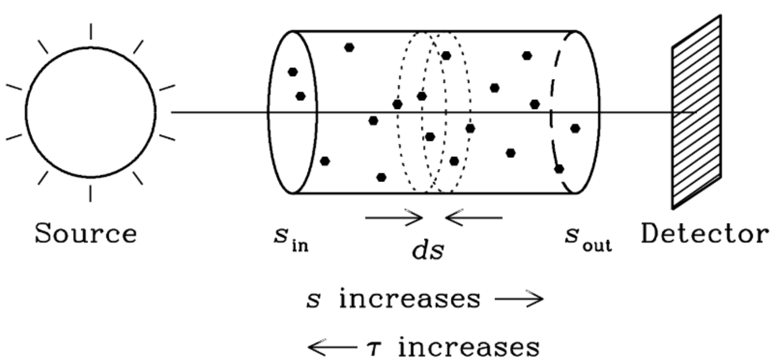


Figure 2: Blackbody curves in linear (left) and log (right)-space. Wien's Law can clearly be seen.

"optical depth" τ is high. To answer this, we have to review a little radiative transfer. The fundamental textbook for RT is "Radiative Processes in Astrophysics" by Rybicki and Lightman.

Radiative transfer

Radiative transfer is the change in intensity dI_ν as radiation propagates from a source to the observer. Along the way, the emission will either be absorbed and scattered by intervening material, or it will encounter an emitting region. Consider an observation of a source shown schematically below (figure taken from "Essential Radio Astronomy") where the specific intensity is modified by absorption (attenuation) and/or emission from an intervening medium.



For attenuation, we can define a "linear absorption coefficient" κ_ν with units of cm^{-1}. *This is misleading since it contains contributions from both absorption and scattering!*[1] Note that

[1] Absorption and scattering are easy to confuse. The main difference is that in scattering, the scattered

this is not opacity or mass absorption coefficient, although both share the same notation and a similar definition! Sorry for the confusion. The amount of energy absorbed is proportional to the light intensity:

$$dI_\nu = -\kappa_\nu I_\nu ds, \tag{3}$$

where ds is the path. Absorption removes photons from the path, thus the negative sign. It is worth pointing out here that absorption excites atoms and molecules, and these atoms and molecules then re-emit. It this emission was beamed along ds there would be no change in intensity. Instead, the re-emitted light is more generally close to isotropic, so the emission is reduced.

For emission, we can define the emission coefficient j_ν as:

$$dI_\nu = j_\nu ds. \tag{4}$$

Notice that there is no dependence on I_ν, in contrast to absorption. The units of j_ν are $\mathrm{erg\,cm^{-1}\,sr^{-1}\,s^{-1}}$.

The total change in intensity is therefore

$$dI_\nu = j_\nu ds - \kappa_\nu I_\nu ds, \tag{5}$$

or

$$\frac{dI_\nu}{ds} = j_\nu - \kappa_\nu I_\nu. \tag{6}$$

Equations 3 and 4 represent one form of the *Equation of Radiative Transfer*. This is one of the fundamental equations in astrophysics. All it is saying, however, is that the change in intensity along the path is just the emission (j_ν) minus the absorption ($\kappa_\nu I_\nu$).

Let's take the illustrative example of no emission. In this case

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu, \tag{7}$$

which has a solution

$$I_\nu(s) = I_{\nu,0} e^{-\kappa_\nu s}, \tag{8}$$

where $I_{\nu,0}$ is the unattenuated emission. The radiation intensity will decrease exponentially.

We can also define the dimensionless quantity of *optical depth* τ from

$$d\tau_\nu = -\kappa_\nu \, ds. \tag{9}$$

or

$$\tau_\nu = -\int \kappa_\nu \, ds, \tag{10}$$

radiation direction depends on the incident photon direction. Re-emission follows absorption, but this re-emission is isotropic.

where the integration is carried out over the path length. In most cases, we need only integrate over the source of interest. For example, if there is a gas cloud 20 kpc away that is 1 kpc thick, we may be able to only integrate over the 1 kpc of the cloud if the rest of the 20 kpc can be assumed to have no impact. For completeness,

$$I_\nu(\tau_\nu) = I_{\nu,0} e^{-\tau_\nu}, \tag{11}$$

The optical depth ranges from zero to infinity. Low values $\tau_\nu \ll 1$ are called "optically thin." These are things you can see through *at that particular frequency*. A good example is glass, which has a very low optical depth at optical frequencies, but actually has a high optical depth in the ultra-violet. High values $\tau_\nu \gg 1$ are called *optically thick*. A wall is optically thick at optical frequencies. A wall is optically thin at X-ray frequencies. Near $\tau \simeq 1$ we have to be careful - this is marginally optically thick.

If we rewrite things in terms of the optical depth, using $\frac{d\tau_\nu}{ds} = \kappa_\nu$,

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\kappa_\nu} - I_\nu. \tag{12}$$

We can further define the *Source function* S_ν

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \tag{13}$$

Combining our expressions, we arrive a second form of the *Equation of Radiative Transfer*, this time using the optical depth and source function so that

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu. \tag{14}$$

We will use this one from now on, because optical depth is a much better and more measurable parameter compared to actual linear depth.

In (full) thermodynamic equilibrium (TE) at temperature T, there is no change in intensity along the path and $\frac{dI_\nu}{d\tau} = 0$. In this case, $I_\nu = S_\nu = B_\nu(T)$, our old friend the Planck function. When is $I_\nu = B_\nu(T)$??? When $d\tau \rightarrow \infty$! Or in other words, when the optical depth is high, the intensity is that of a blackbody at temperature T. In this case, nothing else about the source matters, only its temperature.

This is a subtle, but extremely important point. For high optical depth sources, the only emission you can get out is that of a blackbody. You cannot for example get line emission. The source properties, aside from temperature, do not matter. The only thing you see is the *surface* emission. In fact, you only see down on average to the depth where the optical depth is unity. Think of a wall again, where you cannot determine how thick it is since you only see the paint layer (ok, so a wall actually is not a perfect blackbody since paint reflects light of different wavelengths....). Contrast this with glass. As glass get thicker, and thicker, we will notice more of a green hue. By determining *how* green it is, we can work out how thick it is. We will return to this point later.

Solutions to the Equation of Radiative Transfer

The deceptively simple equation of radiative transfer has had volumes written about its solutions. We can integrate the transfer function by multiplying by e^{τ_ν}. If we define $\tau_\nu = 0$ at $I_{\nu,0}$, we find

$$I_\nu(\tau_\nu) = I_{\nu,0}(\tau_\nu)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau')e^{-(\tau_\nu-\tau'_\nu)}\,d\tau' \tag{15}$$

The intensity I_ν at optical depth τ_ν is the initial (background) intensity $I_{\nu,0}$ attenuated by a factor $e^{-\tau_\nu}$, plus the emission $S_\nu d\tau^{'}$ integrated over the path, itself attenuated by the factor $e^{\tau_\nu-\tau_\nu^{'}}$. This final exponent represents "self-absorption." The material itself will absorb its own radiation. "Self-absorption" refers to absorption by one species (HI, CO, etc) by that species. If background radiation from e.g. HI is absorbed by optically thick HI, this is called self-absorption. This is known as the "formal solution to the equation of radiative transfer."

The difficulty in using Equation 15 is that in general we don't know how S varies with τ, because S depends on I, which is not known until S is known. It's a circular problem, which is why it is often solved computationally. It is worth examining this equation a bit more in limiting cases that allow us to simplify the integral:

$\boldsymbol{\tau = 0}$

If the optical depth is zero, we get $I_\nu = I_{\nu,0}$, simply the background intensity back. If there is no optical depth, we get neither emission nor absorption (like a window!). This illustrates how emission and absorption are intimately related.

S constant

We can sometimes make the assumption that S_ν is a constant, so we can pull it out of the integral:

$$I_\nu(\tau_\nu) = I_{\nu,0}(\tau_\nu)e^{-\tau_\nu} + S_\nu \int_0^{\tau_\nu} e^{(-\tau-\tau')}\,d\tau' = I_{\nu,0}(\tau_\nu)e^{-\tau_\nu} + S_\nu(1-e^{-\tau_\nu}) \tag{16}$$

The first term on the right hand side is attenuation along the line of sight. The second one is emission along the line of sight.

S constant, LTE

In Local Thermodynamic Equilibrium, LTE, $S_\nu = B_\nu(T)$, so

$$I_\nu(\tau_\nu) = I_{\nu,0}(\tau_\nu)e^{-\tau_\nu} + B_\nu(1-e^{-\tau_\nu}). \tag{17}$$

We will discuss LTE later, but essentially it means that for a small volume we can assume a single temperature that is also reflected in the level populations of the atoms and molecules.

S constant, LTE, Radio Regime

In the radio, we use the brightness temperature instead of the intensity. They are related by $I_\nu = \frac{2\nu^2}{c^2} k T_B$. We can also use the Rayleigh-Jeans approximation $B_\nu(T) = \frac{2\nu^2}{c^2} kT$, with T here the kinetic temperature. Since these relationships both have the same constants, we can write

$$T_B = T_{B,0} e^{-\tau_\nu} + T(1 - e^{-\tau_\nu}). \tag{18}$$

Note that the use of the Rayleigh-Jeans approximation here does not imply that the material is optically thick. It just implies that the emission is still modified by the optical depth.

S constant, LTE, Optically Thin

If $\tau \ll 1$, we get emission from the background radiation, as well as from along the line of sight. We can make the Taylor expansion substitution $e^{-\tau_\nu} \simeq 1 - \tau_\nu$, so

$$I_\nu(\tau_\nu) = I_{\nu,0}(\tau_\nu)(1 - \tau_\nu) + B_\nu \tau_\nu \simeq I_{\nu,0}(\tau_\nu) + B_\nu \tau_\nu. \tag{19}$$

The first term again is the background radiation attenuated by the ISM. The second term is the Planck function modified by the optical depth of the ISM. Notice that we can still have a blackbody-like spectrum even if it is optically thin, although it is modified by the optical depth (which is less than 1). In the case that $\tau_\nu = 0$, we of course only see the background radiation. In the radio regime,

$$T_B = T_{B,0}(1 - \tau_\nu) + T\tau_\nu \simeq T_{B,0} + T\tau_\nu. \tag{20}$$

S constant, LTE, Optically Thick

If $\tau \gg 1$, $e^{-\tau_\nu} \to 0$, so

$$I_\nu = S_\nu. \tag{21}$$

If there is a blackbody in our line of sight, we don't see any emission from behind it. In radio astronomy, $T_B = T$ for optically thick emission, the kinetic temperature of the material (if in LTE).

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + B_\nu(T)(1 - e^{-\tau_\nu})\,, \tag{22}$$

where $I_\nu(0)$ is the background radiation and τ_ν is the optical depth. So as $\tau \to \infty$, $I_\nu(\tau_\nu) \to 0 + B_\nu(T)(1-0) = B_\nu(T)$. To summarize, the Planck function has units of specific intensity or surface brightness, and in the limit of high optical depth, $I_\nu = B_\nu$.

There are a two important points about blackbody radiation.

1. The wavelength (or frequency) of peak intensity is inversely related to the temperature via Wien's Law:

$$\lambda_{\max} = \frac{0.2898}{T(\mathrm{K})}\ \mathrm{cm}\,, \tag{23}$$

or

$$\nu_{\max} = 5.879 \times 10^{10} T(\,\mathrm{K})\,. \tag{24}$$

We can derive these by setting the differential of B_λ or B_ν equal to zero. This tells us that hotter things peak at shorter wavelengths and higher frequencies.

The Sun for instance is 6000 K and peaks near 500 nm in the optical. A 10^4 K star peaks closer to 300 nm in the UV.

2. A hotter blackbody has a higher surface brightness intensity at *all* frequencies. This can be seen in Figure 2.

   It is important to remember that more intensity at all frequencies does not necessarily mean more energy! Think about burners on a stove. A small hot burner will have very intense radiation. A large cooler burner will have less intense radiation. But the larger one may boil water faster because although its intensity (surface brightness) is lower, it emits more total energy. What matters is the product of the surface brightness and the emitting area.

   Let's quantify this. To find the intensity (not the specific intensity), we integrate over all frequencies or wavelengths:

$$B(T) = \int_0^\infty B_\nu(T) d\nu\,. \tag{25}$$

   After some math, this integral results in the expression

$$B(T) = \frac{\sigma T^4}{\pi}\,, \tag{26}$$

   where σ is of course the Stephan-Boltzmann constant. In the case of an isotropic radiation field, which we can frequently assume, it can be shown that $F_\nu = \pi B_\nu$, so therefore $F = \sigma T^4$. This is of course the *Stephan-Boltzmann Law.* We are often interested in the total luminosity of an object (in erg s^{-1} or W):

$$L = \int_S F dA\,, \tag{27}$$

   the flux integrated over the emitting surface. For spherical objects, this leads to $L = 4\pi r^2 \sigma T^4$, where r is the object's radius. Thus, the total energy output is related to the surface area and the temperature.

Using Blackbodies

We can usually assume that stars emit similarly to blackbodies, in which case we know their approximate spectral shape for a given temperature. Therefore, observations of stars using astronomical filters can give you information about the temperatures of those stars. Since the temperature and mass are related, we can get a proxy for mass.

The flux (or magnitude) that we measure depends on the filter used. In the optical we may use the U, B, and V filters. We measure the convolution of the filter transmittance and the source spectrum.

Imagine two filters placed on a blackbody curve. The flux ratio of these filters will give you some information about how the intensity is changing. For example, if the flux ratio is large (the longer-wavelength filter is reading much less), the decrease is steep and we must be on the long wavelength side of a high temperature peak. If the flux ratio is small, we must be on the short wavelength side of a low temperature peak. From our discussion of magnitudes, we know that flux ratios are called colors. Colors therefore tell you about the spectral shape, and the temperature of the object.

That colors are useful relies on the fact that stellar spectra are similar to that of blackbodies. This is obvious from Figure 3 below (Figure 3.11 in Carroll & Ostlie), where the U-V and B-V colors of stars are compared to those of blackbodies.

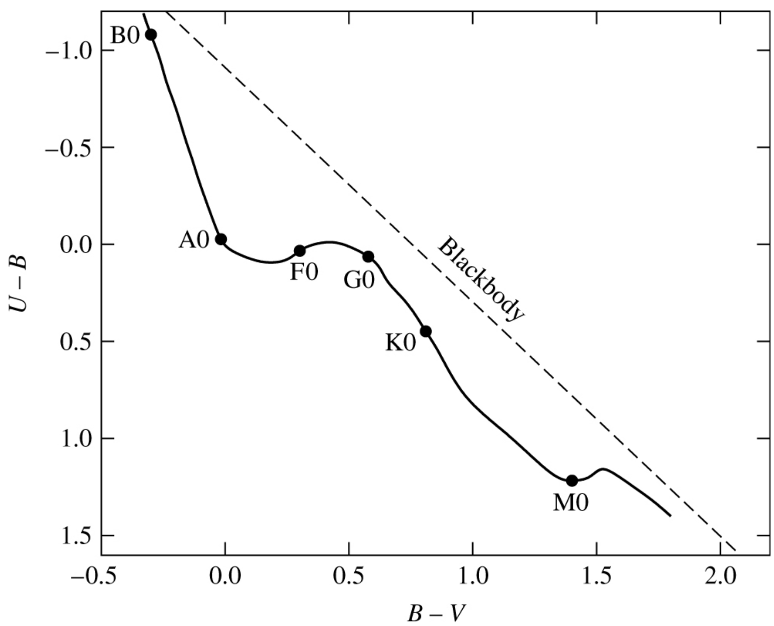


Figure 3: $B - V$ and $U - B$ colors for star of various spectral types. B0 is the largest and M0 are the smallest mass stars in the diagram.

Astronomers use colors as a proxy for temperatures, for example on the color-magnitude diagram, CMD. The CMD looks almost exactly like the H-R diagram because there is such a clean mapping between colors and temperatures. Why use the CMD? The quantities are entirely observable. In the H-R diagram, we often do not know the luminosity and temperature, but we can easily measure magnitudes for a bunch of stars.

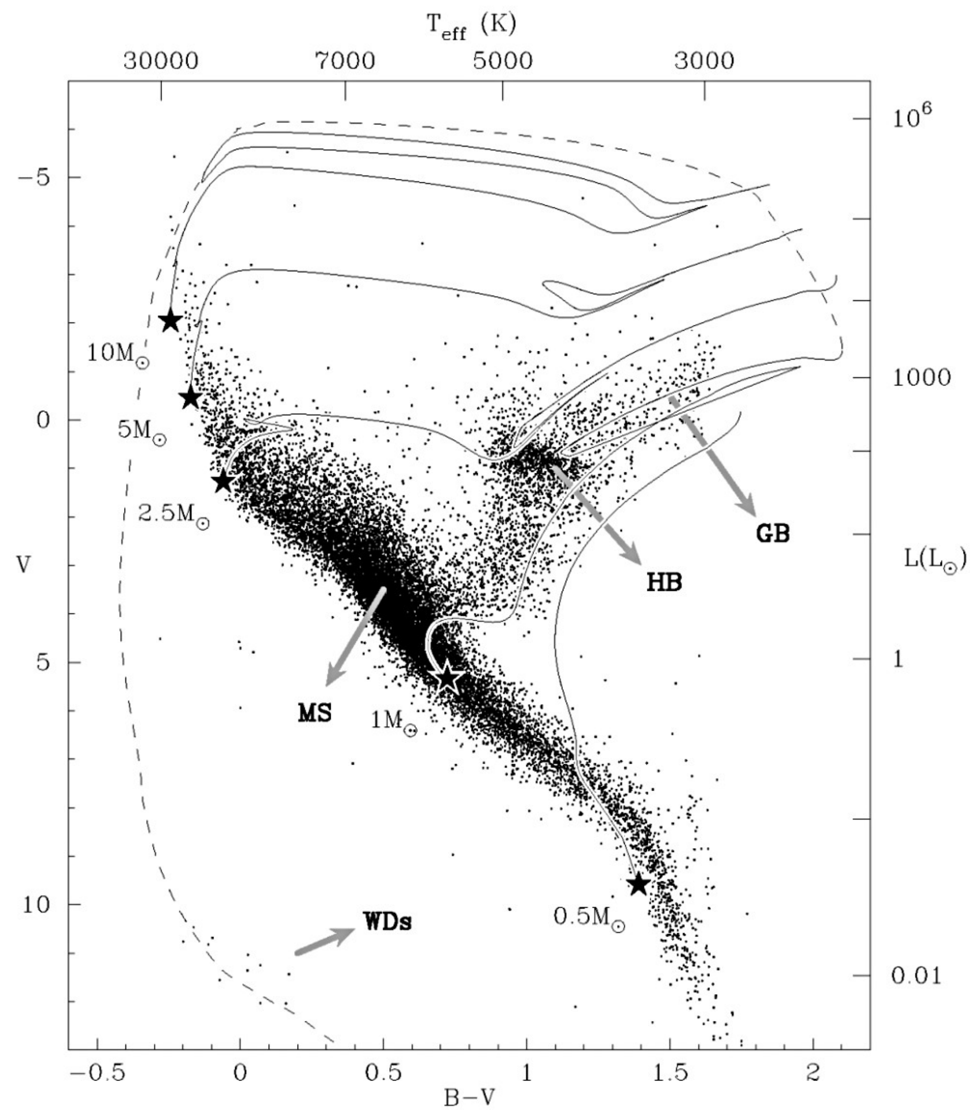


Figure 4: A Color-Magnitude Diagram (CMD). Each dot corresponds to one star. Shown are the main sequence (MS), location of white dwarfs (WD), the Horizontal Branch (HB), and the Giant Branch (GB). With time, stars evolve off the main sequence, go up into the giant branch, back down into the horizontal branch, and eventually become white dwarfs. The evolutionary tracks for stars of various masses are also shown.