ASTR705

HW #9

1) $R_S = \left(\frac{3}{4\pi} \frac{Q_0}{n^2 \alpha_B} \right)^{1/3}$

Assume $n = 100\ cm^{-3}$, $\alpha_B = 7.0 \times 10^{-13} T_4^{-3/4}$, $T_4 = 1$

$\log(Q_0) = 49.64, 48.99, 47.88$ for O3, O6, O9.5

$\log(Q_1) = 49.04, 48.76, 45.80$

$R_i = R_S \left(1 + \frac{7}{4} \frac{ct}{R_S} \right)^{4/7}$

$c = \left(\frac{\gamma kT}{m} \right)^{1/2} = 1.17 \times 10^6\ cm/s$ for $\gamma = 5/3$, $T = 10^4 K$, $m = m_H$

$R_{i,He} = R_S \left[\frac{0.68\ Q_1/Q_0}{1 - 0.17\ Q_1/Q_0} \frac{n_H}{n_{He}} \right]^{1/3}$

Assume $n_H/n_{He} = 10$

2) $\frac{I(OII\ 4364\text{Å})}{I(OIII\ 5008\text{Å})} = 0.003$

$\frac{I(OII\ 3729\text{Å})}{I(OII\ 3729\text{Å})} = 1.2$

c) Appendix D OI → OII 13.6eV

OII → OIII 35.1eV

b) From Figure 18.7, $T_e \sim 8000K$

From Figure 18.4, $T_4^{-1/2} n_e \sim 240\ cm^{-3}$

So $n_e \sim 240 \cdot 0.8^{1/2} \sim 215\ cm^{-3}$

c) $\frac{I_1}{I_2} = 10^{0.4(m_2 - m_1)}$

If $A \propto 1/\lambda$, $\frac{A_1}{A_2} = \frac{\lambda_2}{\lambda_1} = 0.87$

so if $A_{5008} = 0.5$, $A_{4364} = 0.5/0.87 = 0.57$

Rono changes by $10^{0.4(0.57-0.00)} = 1.07$

Real roho = $0.003 \times 1.07 = 0.0032$

For OII, λ too close to matter

Get the same results!

3) $\frac{T_L}{T_c} = \frac{\tau_L}{\tau_c}$ | $\tau_L \ll 1$

$$\tau_c = 1.091\, T_4^{-1.323}\, \nu_9^{2.118}\, \frac{n_1}{n_7}\, \frac{EM}{10^{25}\,cm^{-5}}$$

$$\tau_L = 1.92\times10^{3} \left(\frac{T_e}{K}\right)^{-5/2} \frac{EM}{cm^{-6}\,pc} \left(\frac{\Delta\nu}{kHz}\right)^{-1}$$

$$\frac{\Delta\nu}{\nu} = \frac{\Delta v}{c}$$

$$\frac{\tau_L}{\tau_c} = \frac{1.92\times10^{3} \left(\frac{T_e}{K}\right)^{-5/2} \left(\frac{EM}{cm^{-6}\,pc}\right) \left(\frac{\Delta v}{c}\,\frac{\nu}{kHz}\cdot\frac{10^6 kHz}{GHz}\right)^{-1}}{1.091 \left(\frac{T_e}{K}\right)^{-1.323} \cdot \left(\frac{1}{10^9}\right)^{-1.323} \left(\frac{\nu}{GHz}\right)^{2.118} \frac{EM}{10^{25}\,cm^{-5}}\, 3.08\times10^{18}\,\frac{cm}{pc}}$$

$$= C \left(\frac{T_e}{K}\right)^{1.177} \left(\frac{\nu}{GHz}\right)^{1.118} \left(\frac{\Delta v}{km/s}\right)$$

$$C = \frac{1.92\times10^{3}}{1.091} \left(\frac{10^6}{3\times10^5}\right) \frac{1}{(10^9)^{1.323}\cdot 10^{-25}\, 3.08\times10^{18}} = 8751$$

$$\frac{T_e}{K} = \left[\frac{\nu^{1.118}}{GHz} \left(\frac{\Delta v}{km/s}\right) \left(\frac{T_L}{T_c}\right)^{-1}\right]^{1/1.177} \cdot 8751$$

$[km/s]$