Nuclear Fusion

1) What determines the rate at which stars create energy?
2) What determines the elements involved?

Fusion energy

Terms:

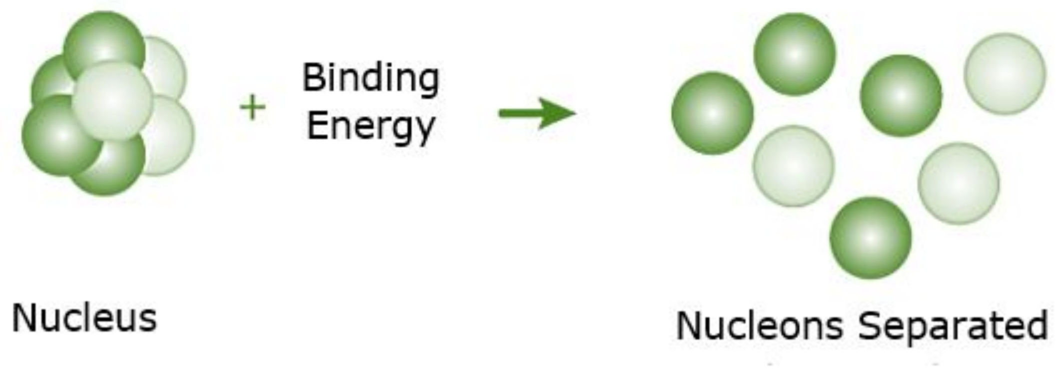
q - the rate of energy release per unit mass

Q_{ijk} - the amount of energy released (with reactants i,j,k)

R_{ijk} - the reaction rate

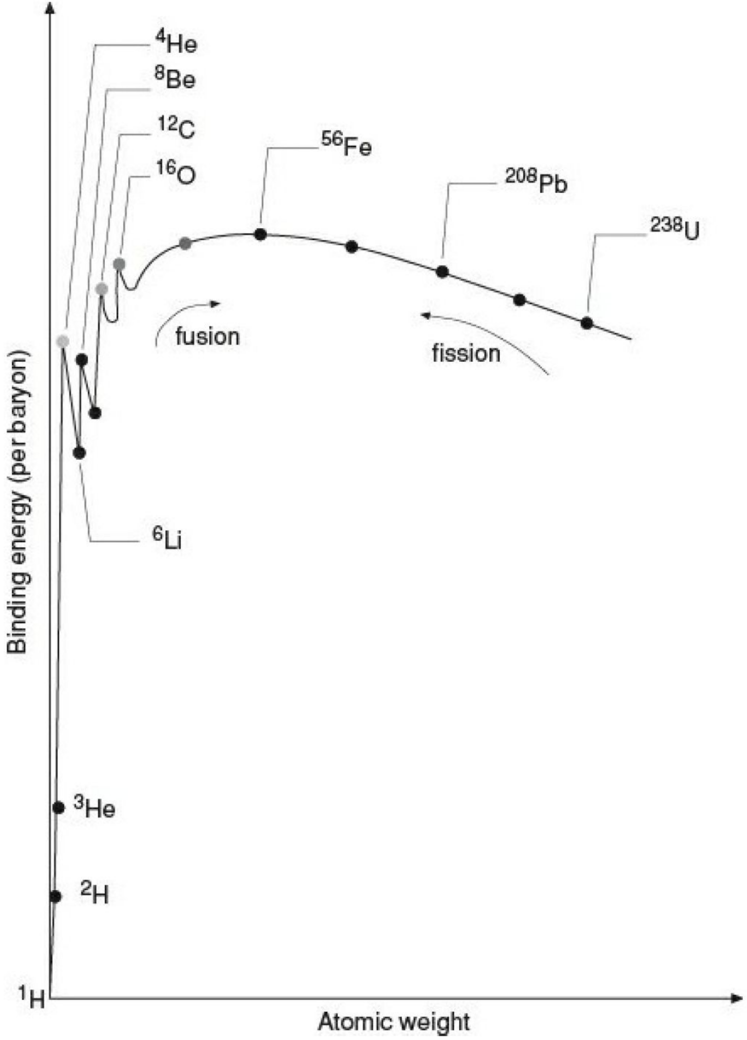
Binding Energy

The amount of energy required to separate a particle from a system of particles or to disperse all the particles of the system.



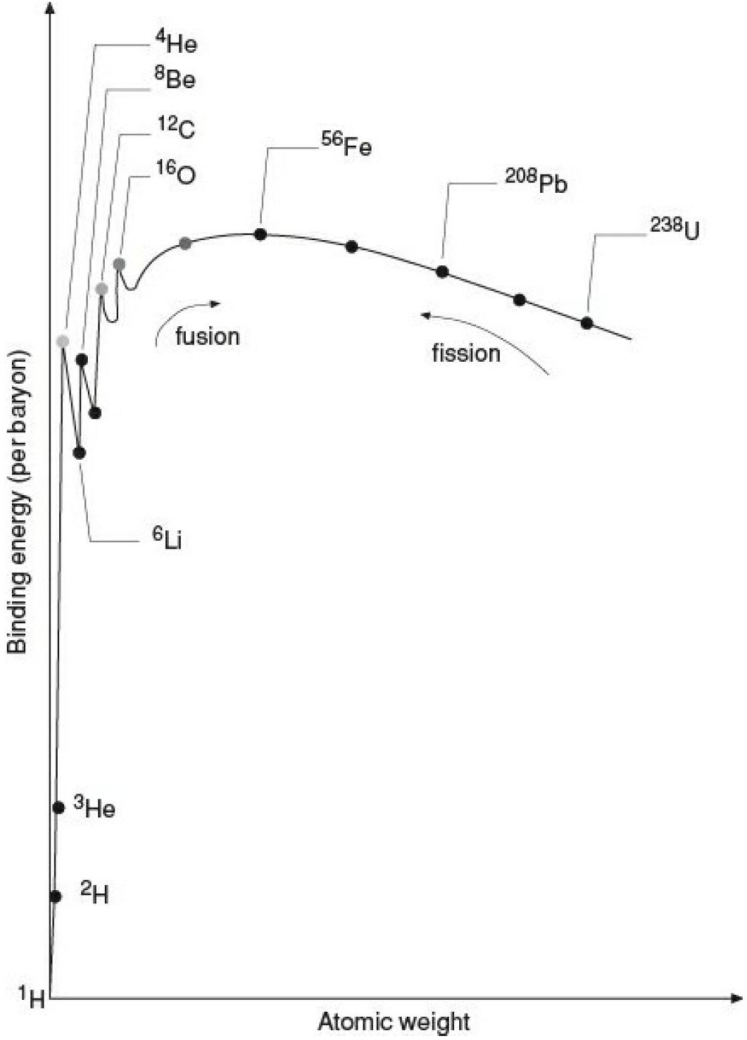
Binding energy per nucleon

Note: There are no stable configurations for A=5 or A=8



Binding energy per nucleon

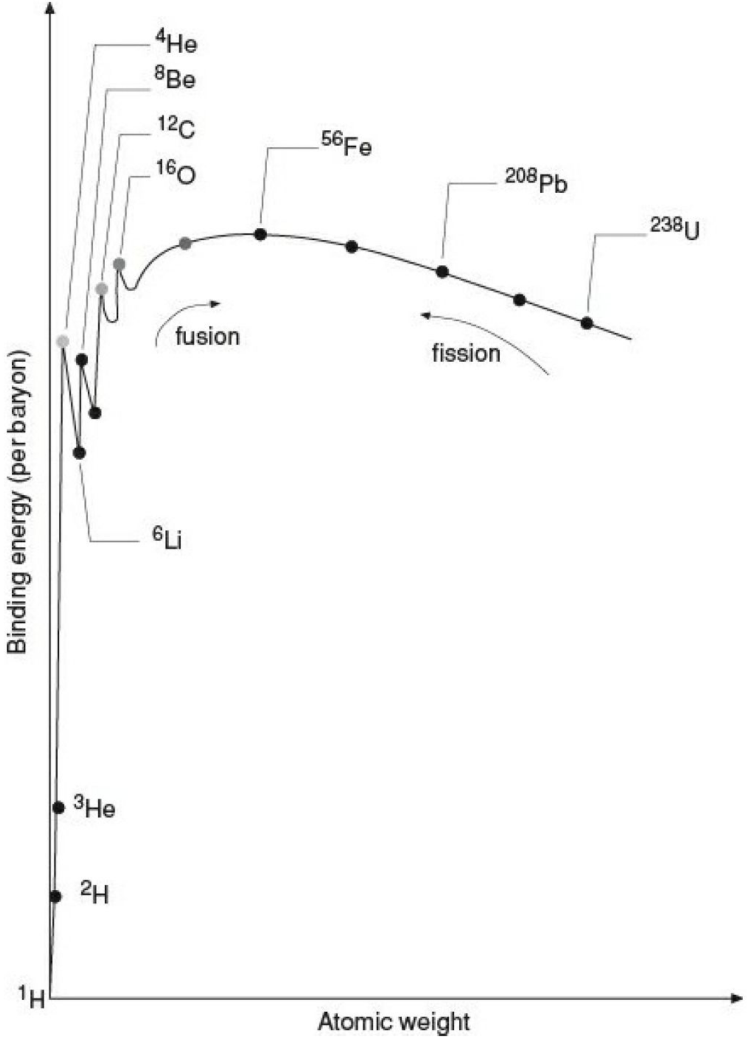
“The energy released in a nuclear reaction Q_{ijk} is a measure of the difference between the binding energies of the reactants and the products.”



Binding energy per nucleon

A few more things:

1) He^4 is higher than elements around it
2) Where there is a sharp rise, fusion of similar nuclei will produce a lot of energy
3) The peak of the curve is iron.



Reaction Rates

How can we determine a nuclear reaction rate?

Reaction Rates

For a single particle in a field of motionless particles, $R=\sigma v$ (book uses ς)

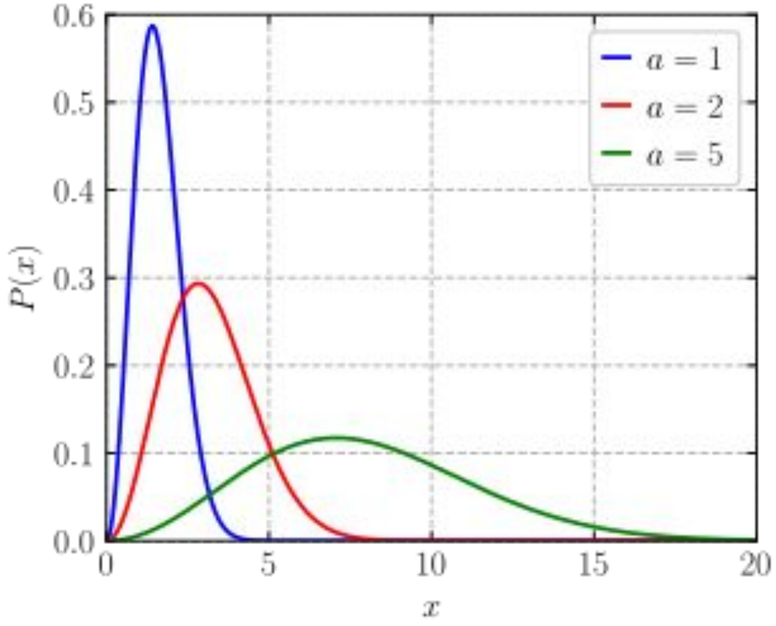
What is v?

Reaction Rates

For a single particle in a field of motionless particles, R~1/(nσv)

What is v?

A Maxwellian, so v $\propto$ exp(-mv^2/2kT)



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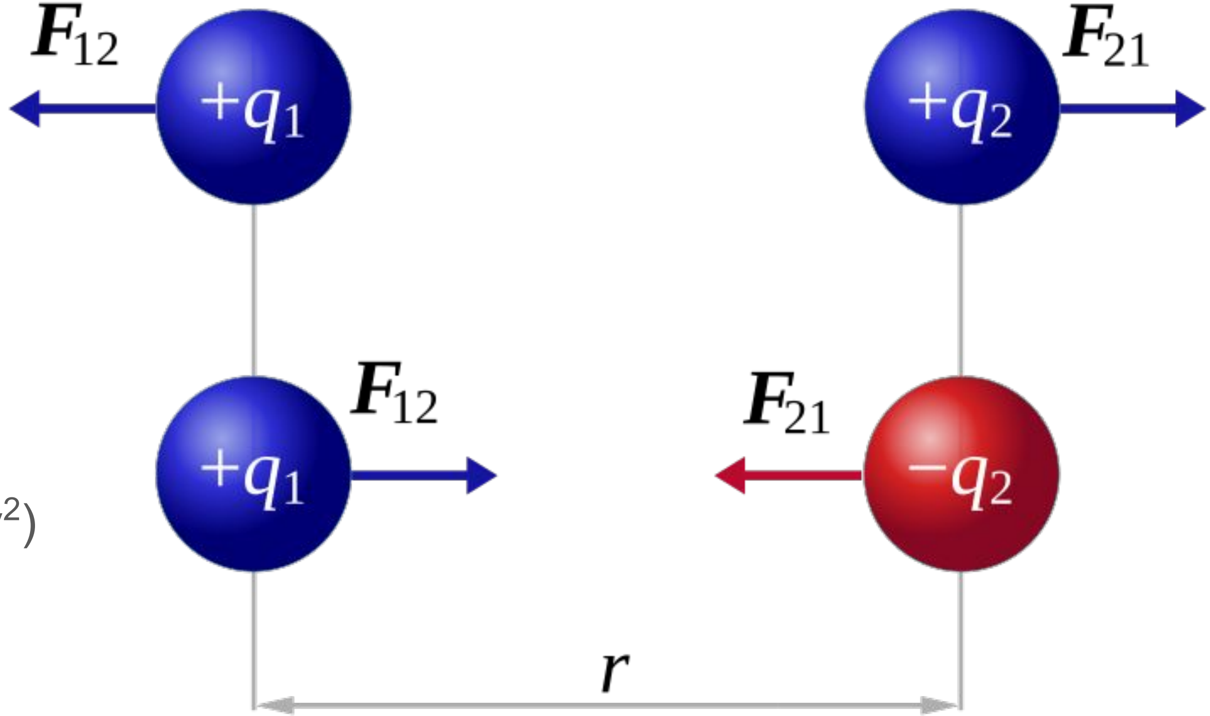
But cross section is harder….

Coulomb Barrier

$\frac{1}{2} m_g v^2 > Z_i Z_j e^2/(4\pi\varepsilon_0 r)$

So effective “barrier”:

$d = 1/(4\pi\varepsilon_0)\, Z_i Z_j e^2/\, (\frac{1}{2} m_g v^2)$



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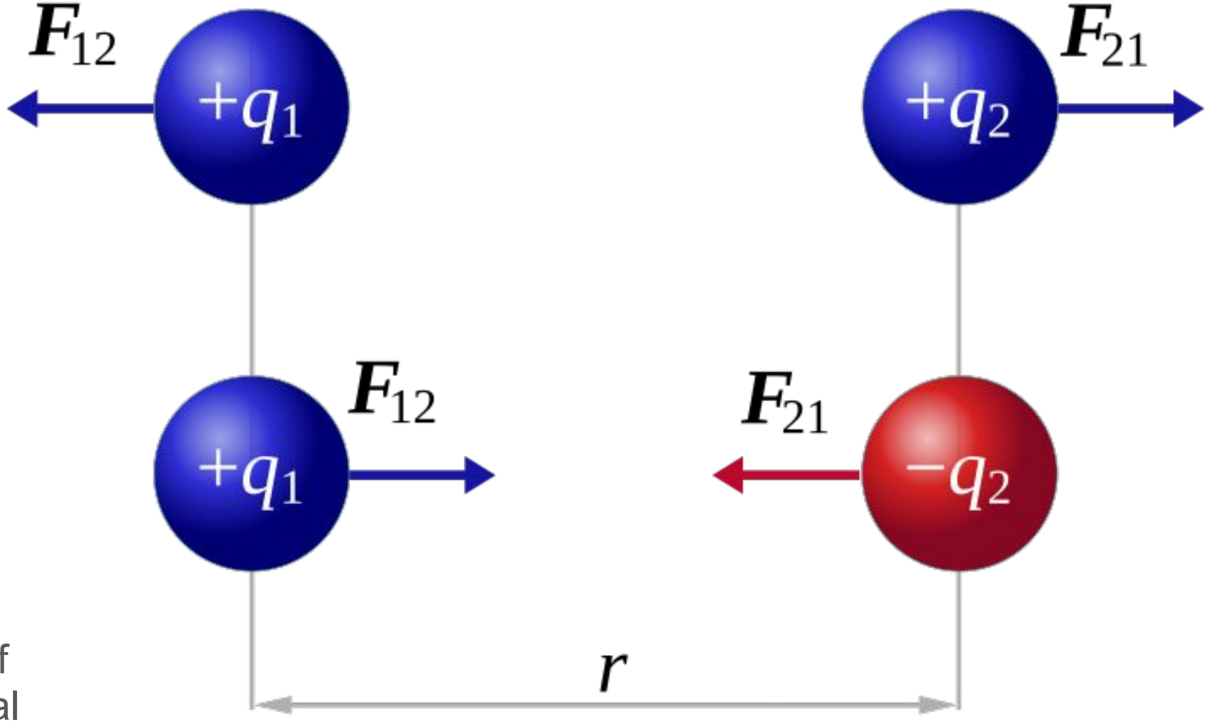
For typical temperatures (and velocities), barrier is 3 orders of magnitude more than the typical range of the strong force!



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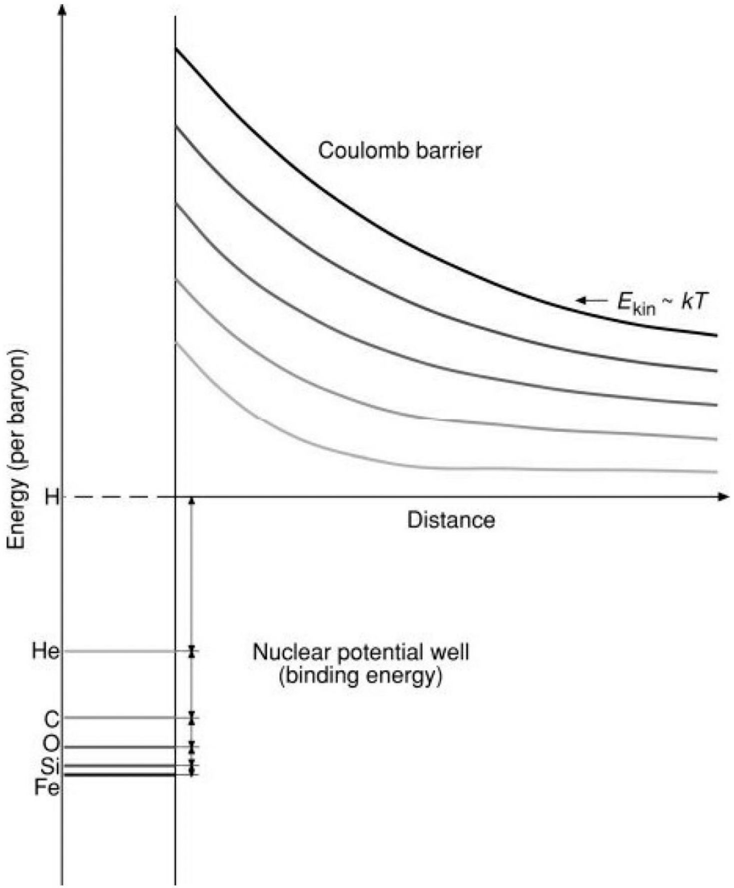

Figure 4.2 Schematic representation of the Coulomb barrier – the repulsive potential encountered by nucleus in motion relative to another – and the short-range negative potential well that is due to the nuclear force. The height of the barrier and the depth of the well depend on the nuclear charge (atomic number).

Gamow peak

The solution is “tunnelling”! George Gamow calculated the tunnelling probability and found that the cross section is proportional to

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Gamow peak

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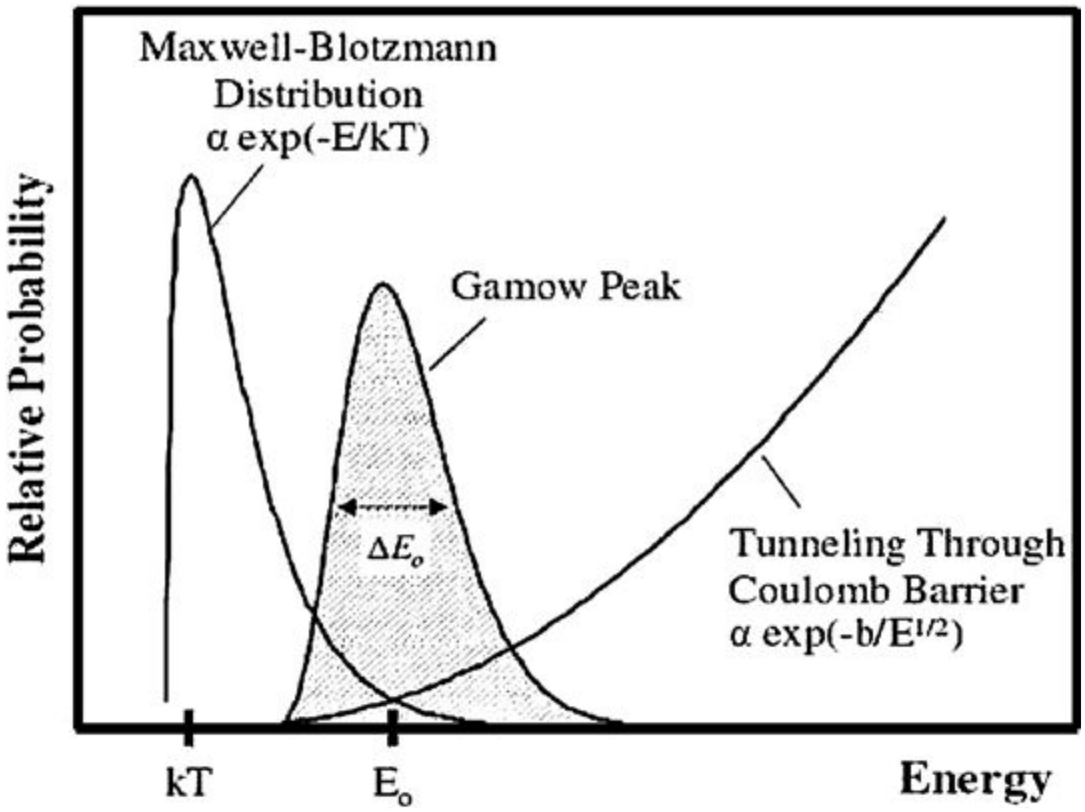
So the rate is proportional to the cross section times the velocity

$$\exp(-\pi Z_i Z_j e^2 / \varepsilon_0 h v) \exp(-m_g v^2 / 2kT),$$

What does this product look like?


Maxwell-Blotzmann
Distribution
α exp(-E/kT)
Gamow Peak
Relative Probability
ΔE_o
Tunneling Through
Coulomb Barrier
α exp(-b/E^1/2)
kT
E_o
Energy

To calculate the reaction rate, we would have to integrate the product over all velocity values. It can be shown that the value of the integral, and with it the reaction rate, is proportional to the maximum of the product, which occurs for

$$v = (\pi Z_i Z_j e^2 kT/\varepsilon_0 h m_g)^{1/3}. \qquad (4.6)$$

Hence the reaction rate

$$\varsigma v \propto (kT)^{-2/3} \exp\left[-\frac{3}{2}\left(\frac{\pi Z_i Z_j e^2}{\varepsilon_0 h}\right)^{2/3}\left(\frac{m_g}{kT}\right)^{1/3}\right]$$

increases with increasing temperature and decreases with increasing charges of the interacting particles. Fusion of heavier and heavier nuclei would therefore require higher and higher temperatures

Timescales

A typical timescale for a reaction is $\tau = 1/(n\sigma v) = 1/(\sigma R_{ijk})$

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A typical timescale for a reaction is $\tau = 1/(n\sigma v) = 1/(nR_{ijk})$

From the earlier relations, this has a very strong dependence on temperature. We can write

$$q = q_0 \rho T^n .$$

(*q* - the rate of energy release per unit mass)

Vocab

Nucleosynthesis - creation of elements via fusion

ignition or threshold temperature - that which is required for a given fusion reaction

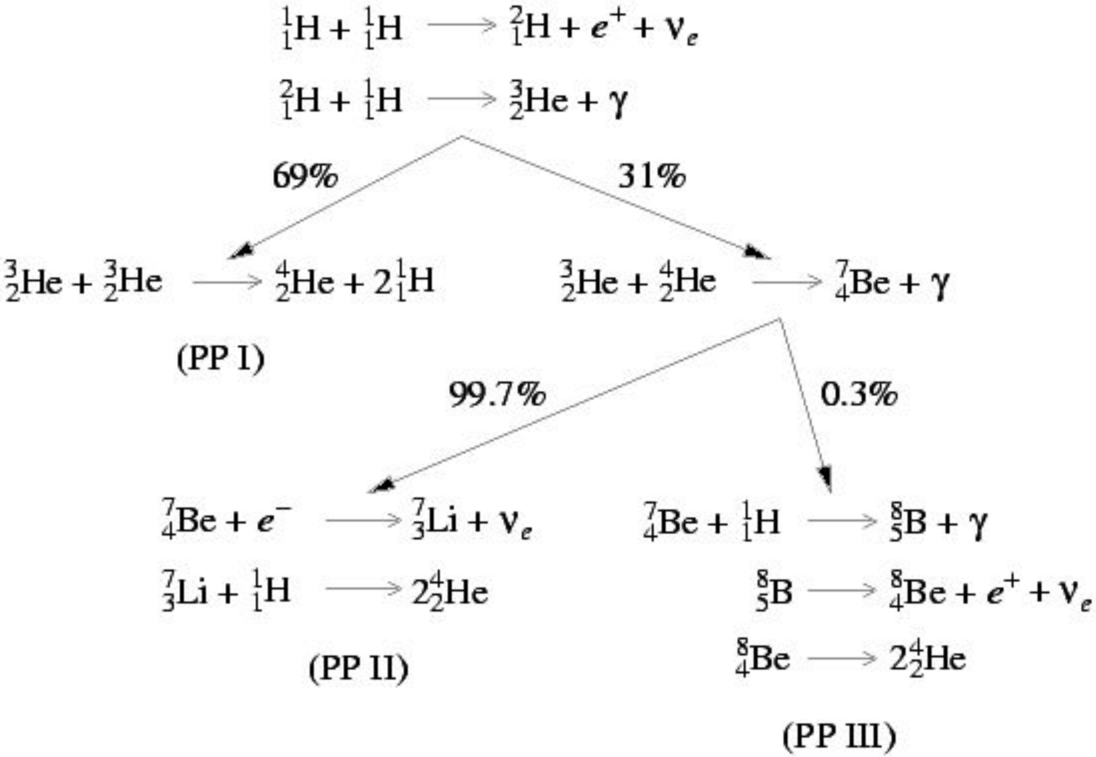
1) What determines the rate at which stars create energy?
2) What determines the elements involved?

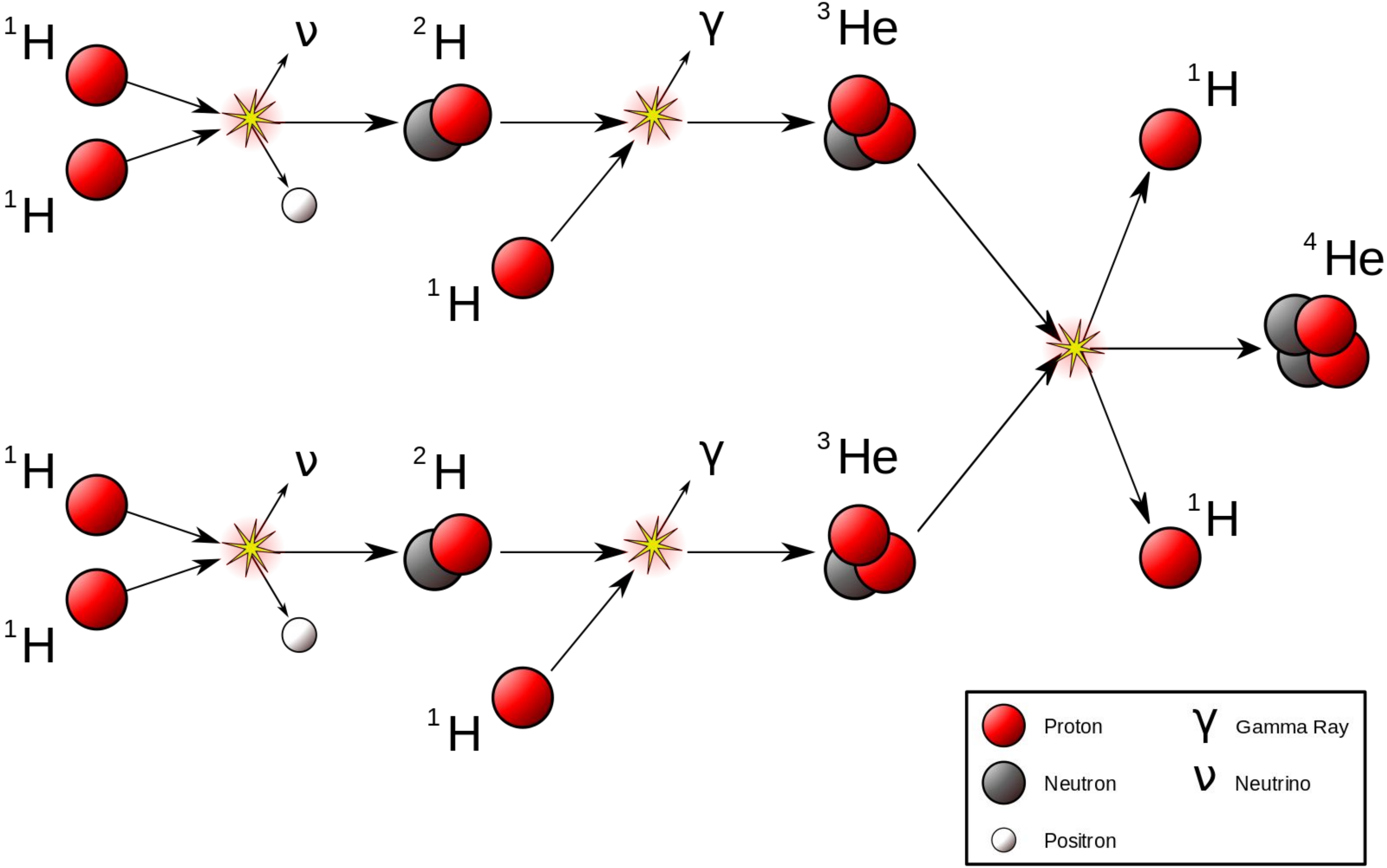
1) What determines the rate at which stars create energy?

   Density (to 1st power) and temperature (possibly to a higher power). Both of these are determined by mass

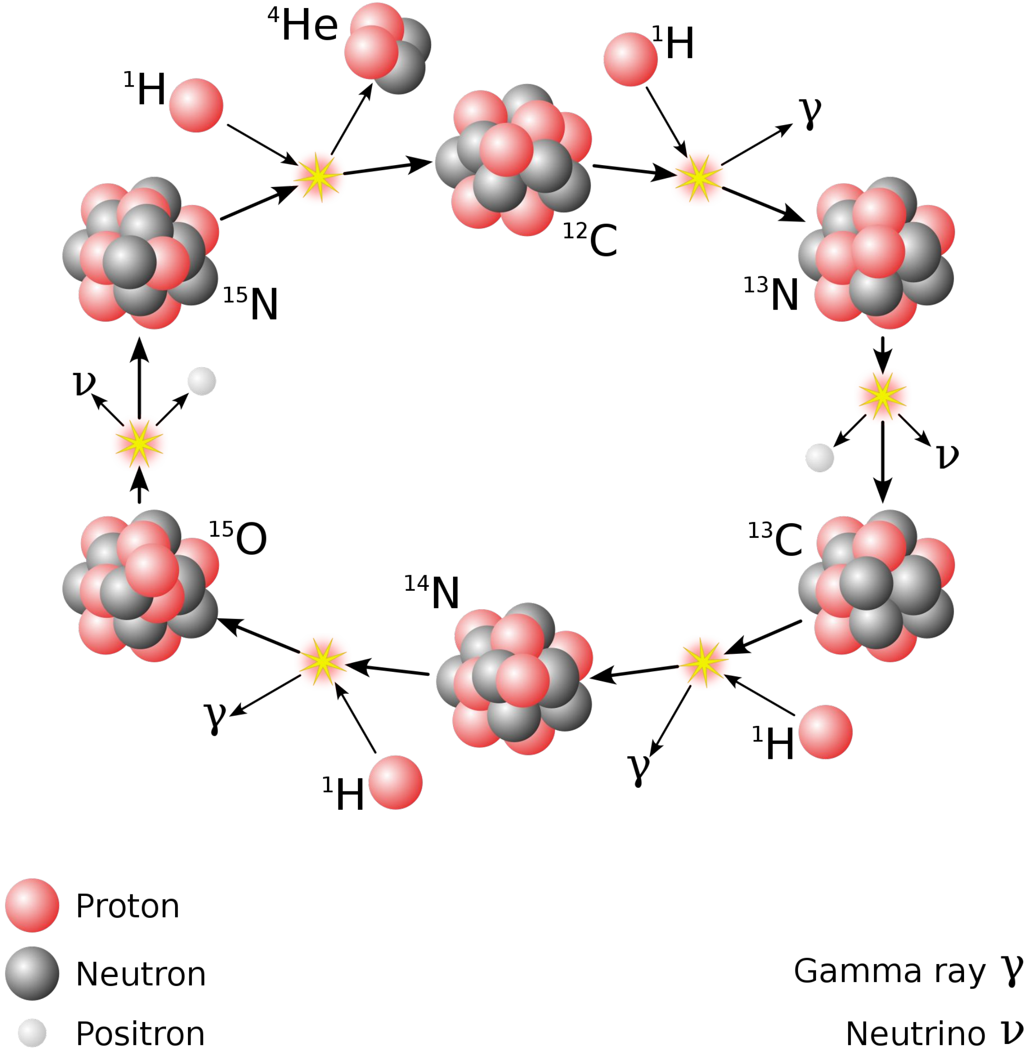
2) What determines the elements involved?

   The difference in binding energy! Differences are largest for the smallest elements.

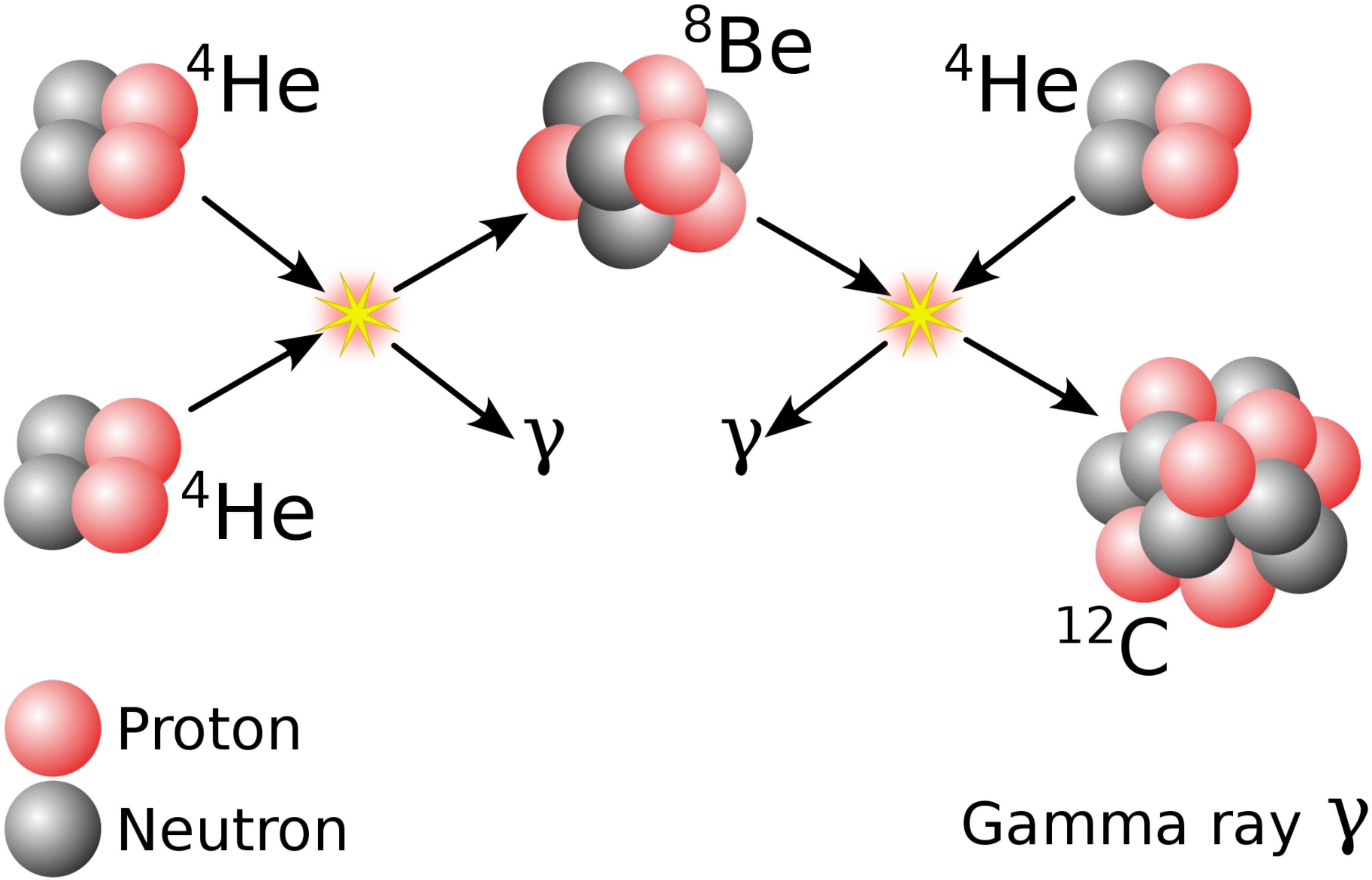

$^1_1H + ^1_1H \longrightarrow ^2_1H + e^+ + \nu_e$
$^2_1H + ^1_1H \longrightarrow ^3_2He + \gamma$
69%
31%
$^3_2He + ^3_2He \longrightarrow ^4_2He + 2^1_1H$
(PP I)
$^3_2He + ^4_2He \longrightarrow ^7_4Be + \gamma$
99.7%
0.3%
$^7_4Be + e^- \longrightarrow ^7_3Li + \nu_e$
$^7_3Li + ^1_1H \longrightarrow 2^4_2He$
(PP II)
$^7_4Be + ^1_1H \longrightarrow ^8_5B + \gamma$
$^8_5B \longrightarrow ^8_4Be + e^+ + \nu_e$
$^8_4Be \longrightarrow 2^4_2He$
(PP III)



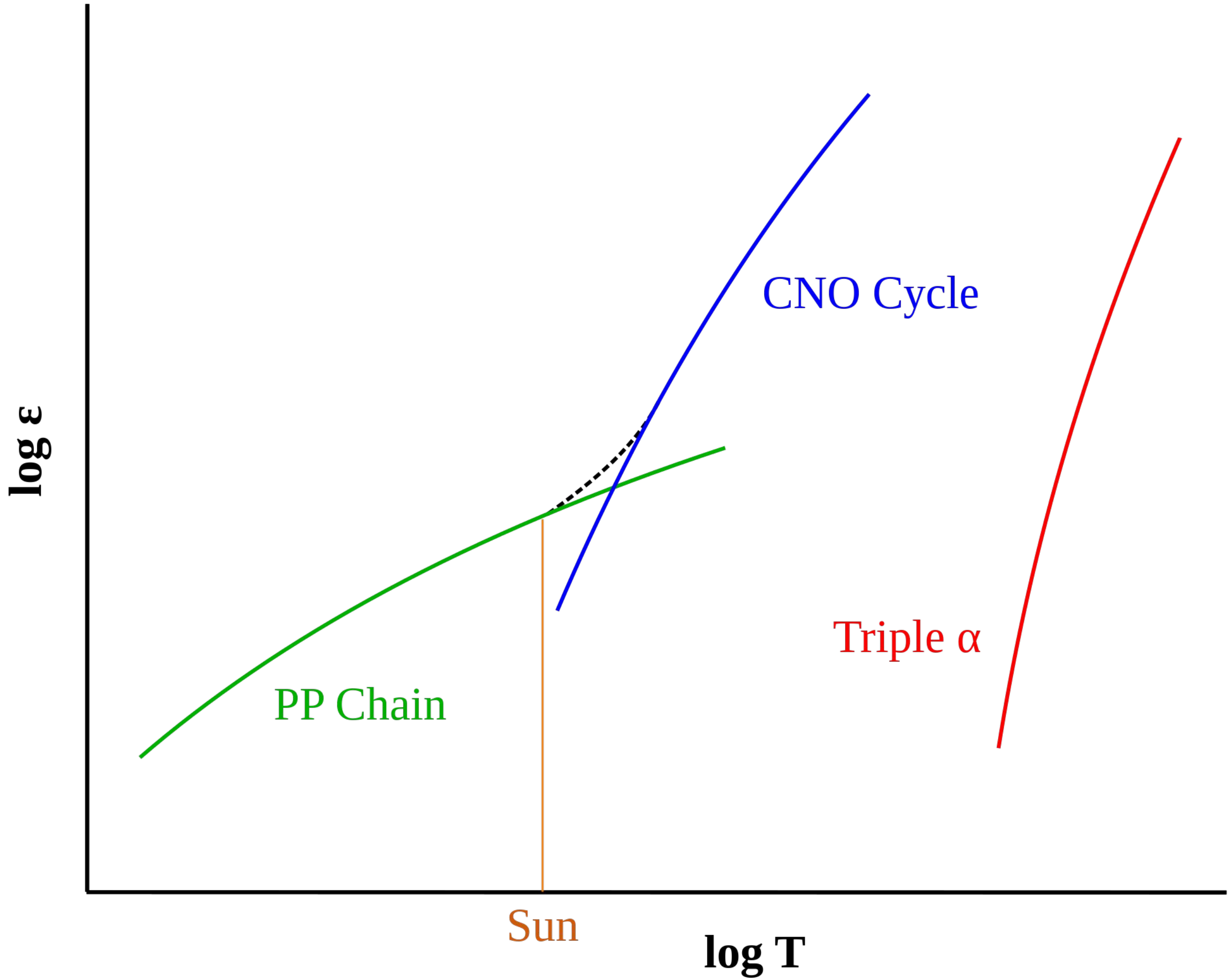
¹H
¹H
ν
²H
¹H
γ
³He
¹H
⁴He
¹H
¹H
ν
²H
¹H
γ
³He
¹H
Proton
Neutron
Positron
γ Gamma Ray
ν Neutrino



^{4}He
^{1}H
^{1}H
γ
^{12}C
^{15}N
^{13}N
ν
ν
^{15}O
^{13}C
^{14}N
γ
γ
^{1}H
^{1}H
Proton
Neutron
Positron
Gamma ray γ
Neutrino ν

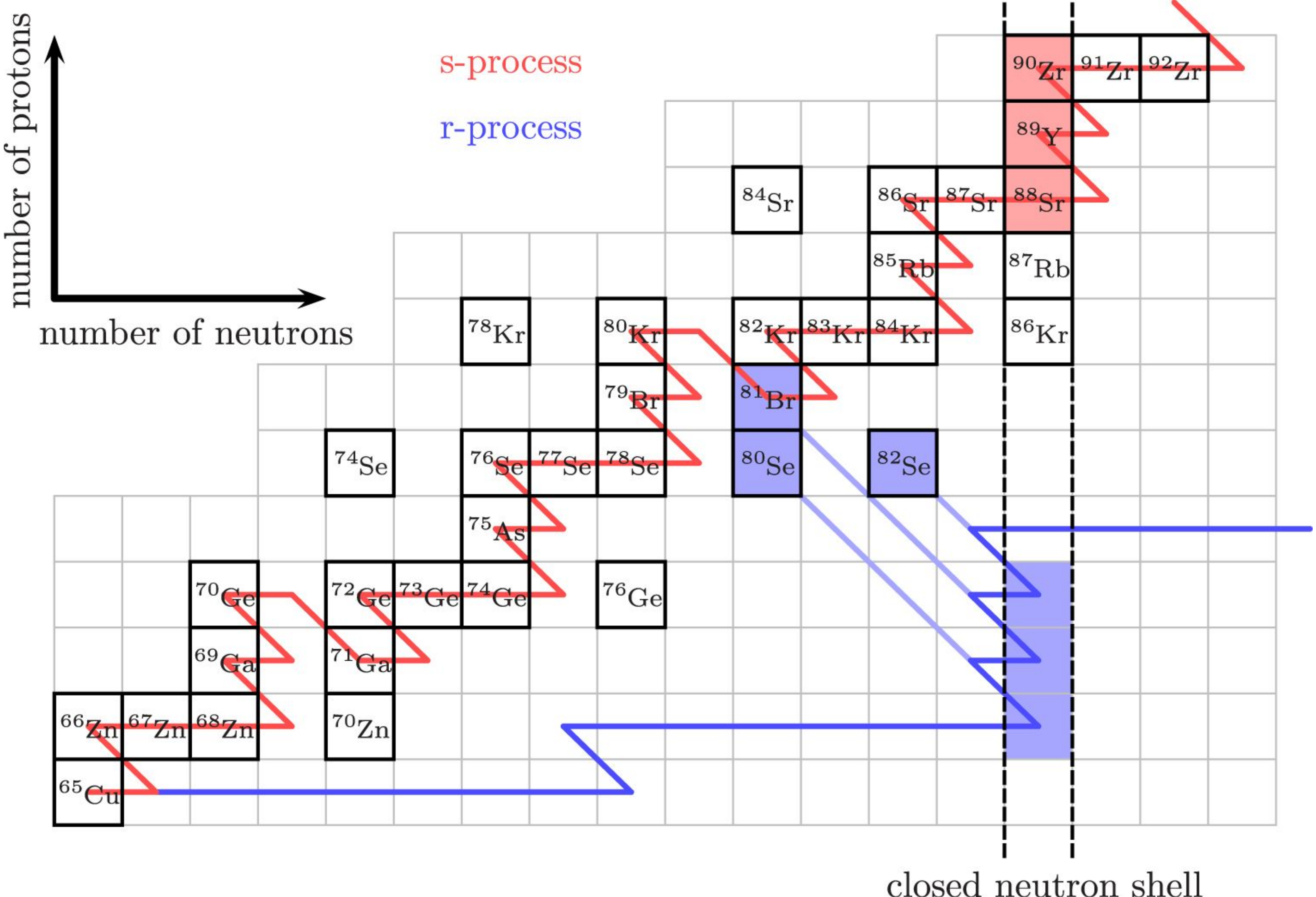


^4He
^8Be
^4He
^4He
γ
γ
^{12}C
Proton
Neutron
Gamma ray γ



log ε
CNO Cycle
Triple α
PP Chain
Sun
log T

Nuclear Fuel	Process	$T_{threshold}$ 10^6K	Products	Energy per nucleon (Mev)
H	PP	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C,O	0.61
C	C+C	600	O,Ne,Ma,Mg	0.54
O	O+O	1000	Mg,S,P,Si	~0.3
Si	Nuc eq.	3000	Co,Fe,Ni	<0.18



s-process
r-process
number of protons
number of neutrons
closed neutron shell
90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
83Kr
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu