

ASTR 705

HW #6

1) $A(\lambda) \propto \lambda^{-1}$

$$R_v = \frac{A_v}{A_B - A_v} = \frac{1/550}{1/440 - 1/550} = \frac{440}{550 - 440} = 4.0$$

$$b) R_v = \frac{(1/550)^\beta}{(1/440)^\beta - (1/550)^\beta} = \frac{1}{(550/440)^\beta - 1}$$

$$\Rightarrow \beta = \frac{\ln(1 + R_v^{-1})}{\ln(550/440)} = 1.75$$

c) High R_v = dense environments, so
probably diffuse environments

$$3) \frac{1}{2} I \omega^2 = \frac{3}{2} kT \rightarrow \omega = \left(\frac{3kT}{I} \right)^{1/2}$$

$$I = \frac{2}{5} m r^2 = \frac{2}{5} \cdot 5 \cdot \frac{4}{3} \pi \cdot 0^3 \cdot r^2 = \frac{8}{15} \pi \cdot 5 \cdot 0^5$$

spheres

$$\nu = \frac{\omega}{2\pi} = \left(\frac{3kT}{\frac{8}{15} \pi \cdot 5 \cdot 0^5} \right)^{1/2} \cdot \frac{1}{2\pi}$$

$$= \left(\frac{45 kT}{8 \pi \cdot 5 \cdot 0^5} \cdot \frac{1}{4\pi^2} \right)^{1/2}$$

$$= \left(\frac{45 kT}{32 \pi^3 \cdot 5 \cdot 0^5} \right)^{1/2}$$

$$b) \nu = 30 \cdot 10^9 \text{ Hz} \quad \rho = 2000 \text{ kg m}^{-3} \quad T_d = 50 \text{ K}$$

$$0^{5/2} = \frac{1}{\nu} \cdot \left(\frac{45 kT}{32 \pi^3 \rho} \right)^{1/2}$$

$$\alpha = \nu^{-2/5} \left(\frac{45 kT}{32 \pi^3 \rho} \right)^{1/5}$$

$$= \left(30 \cdot 10^9 \right)^{2/5} \left(\frac{45 \cdot 1.38 \cdot 10^{-23} \cdot 50}{32 \pi^3 \cdot 2000} \right)^{1/5}$$

$$= 4.4 \text{ \AA}$$

4) Assume $\lambda_m = 100 \mu\text{m} \rightarrow T = 30 \text{K}$

$$b) M = \frac{R F_\nu D^2}{\mu_\nu B_\nu(T, D)}$$

Assume $R = 100$

$$D = 20 \text{ kpc} = 6.16 \times 10^{22} \text{ cm}$$

$$\mu_\nu = \mu_0 \left(\frac{\nu}{\nu_0} \right)^\beta, \quad \beta = 2.0, \quad \mu_0 = 7.3 \text{ cm}^2/\text{g},$$
$$\nu_0 = c/350 \mu\text{m}$$

$$\mu_{100 \mu\text{m}} = 7.3 \left(\frac{350}{100} \right)^2 = 89.4 \text{ cm}^2/\text{g}$$

$$F_{100 \mu\text{m}} = 78 \text{ Jy} = 78 \times 10^{-23} \text{ erg/s/cm}^2/\text{Hz}$$

$$B_{100 \mu\text{m}}(30 \text{K}) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$= 3.36 \times 10^{-12} \text{ erg/s/cm}^2/\text{Hz}$$

$$\Rightarrow M = 9.85 \times 10^{35} \text{ g} = 492 M_\odot$$

