ASTR368
HW#6
March 8, 2024
Due March 22, 2024
2 points each part unless otherwise stated

1) To illustrate the cosmological principle, draw a sample of space that is isotropic (same in all directions), but not homogeneous (the same in all locations). Do the same for a sample that is homogeneous, but not isotropic.

2) What is the implication for the fate of the Universe if the density parameter value $\Omega < 1$?

3) Lifetime of the Universe.
a) Take

$$H^2[1-\Omega]R^2 = -kc^2\,. \tag{1}$$

What is this expression in the present day?
b) Plug in for H and Ω to get an expression of dR/dt, ρ_0, R, and k. Remember that for this Universe with only pressureless dust $R^3\rho = \rho_0$.
c) Solve your answer for a) to derive R as a function of time t by assuming our flat pressureless dust Universe with $k = 0$, $\rho = \rho_c$, and $\Omega = 1$. Your answer will have $\rho_{c,0}$ in it.
d) Substitute in this expression

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \tag{2}$$

and

$$t_H \equiv \frac{1}{H_0} \tag{3}$$

to get an expression for R as a function of t that only depends on t_H.
e) What is the age of this Universe in terms of t_H?

4) Let's again take:

$$H^2[1-\Omega]R^2 = -kc^2 \tag{4}$$

If we also have a radiation energy density Ω_{rad}, where would that go in the equation? What effect would the addition of $\Omega_{\rm rad}$ have on the evolution of the Universe?