ASTR 367
Units and fundamental quantities in astronomy (Chapter 1)

Astronomy has a bunch of units that are strange. The goal of these unusual units is to turn values that otherwise would be very large or very small into numbers that are easier to use and remember. Some units, however, are difficult to get used to, like the choice of the "CGS" system in place of SI.

CGS - let's get it over with

The CGS system is kind of stupid. It stands for "centimeter, gram, second." This would be fine, but using cm breaks the nice metric rule where quantities are related by multiples of $10^{\pm 3x}$.

As far as I know, the use of cm was adopted by astronomers because volumes in cm^{-3} tend to be reasonable values near unity. That's the only reason I know of.

We'll use both CGS and SI in this course, for maximum confusion. Below are some common conversions (from Wikipedia):

Quantity	CGS Unit	Unit definition	Equivalent in SI units
length	cm	1/100 of meter	$= 10^{-2}$ m
mass	g	1/1000 of kilogram	$= 10^{-3}$ kg
time	s	1 second	$= 1$ s
velocity	cm/s	cm/s	$= 10^{-2}$ m/s
acceleration	gal	cm/s^2	$= 10^{-2}$ m/s^2
force	dyn	g cm/s^2	$= 10^{-5}$ N
energy	erg	$\text{g cm}^2/\text{s}^2$	$= 10^{-7}$ J
power	erg/s	$\text{g cm}^2/\text{s}^3$	$= 10^{-7}$ W
pressure	Ba	$\text{g/(cm s}^2)$	$= 10^{-1}$ Pa

Your book as a more extensive list in Appendix B.

Astronomy-Specific Units

Astronomers like to work in units that make the numbers easier to deal with. Common values are for the Sun.

Mass: $M_\odot = 1.9891 \times 10^{30}\,\text{kg} = 1.9891 \times 10^{33}\,\text{g}$

Radius: $R_\odot = 6.955 \times 10^{8}\,\text{m} = 6.955 \times 10^{10}\,\text{cm}$

Luminosity: $L_\odot = 3.84 \times 10^{26}\,\text{W} = 3.84 \times 10^{33}\,\text{erg/s}$ [Note: this is "bolometric" luminosity over all wavelengths]

Distances can get pretty big too, so we'll use the unit of "parsec" (pc). Sometimes astronomical units (AU) are used.

AU $= 1.496 \times 10^{11}$ m $= 1.496 \times 10^{13}$ cm Useful within the Solar system.

pc $= 3.08 \times 10^{16}$ m $= 3.08 \times 10^{18}$ cm Useful for nearest stars

kpc $= 10^3$ pc $= 3.08 \times 10^{19}$ m $= 3.08 \times 10^{21}$ cm Useful for things in the Milky Way and the Local Group.

Mpc $= 10^6$ pc $= 3.08 \times 10^{22}$ m $= 3.08 \times 10^{25}$ cm Useful for external galaxies.

Angles

Everything in astronomy is angles! We often don't know the distance to an object, so instead we measure how large it is on the sky, the angular size. We measure such sizes in degrees, then 1/60 of a degree "minutes," then 1/60 of a minute, "seconds." Because everything is measured on the sky, which appears to be a sphere, we instead use "arcminutes" and "arcseconds." We denote these with $'$, and $''$, respectively.

A useful conversion is that there are $206265''$ per radian.

Solid Angle

Specific intensity above included solid angles, which many students haven't yet heard of. A solid angle, measured in dimensionless steradians (sr), is simply a two-dimensional angle. Think of it as a cone spreading out from the center of a sphere to its edge. A solid angle is the area of a unit sphere such that there are 4π sr total on a sphere. The obvious application is the sky. Objects that appear larger on the sky have a larger solid angle.

The mathematical definition is

$$d\Omega = \sin\theta d\theta d\phi \tag{1}$$

or

$$\Omega = \int_S\int \sin\theta d\theta d\phi\,, \tag{2}$$

where Ω is the solid angle, θ and ϕ are angles in spherical coordinates and the integration is over surface S.

For a spherical solid angle, $\theta = \phi$. For spherical solid angles with small θ we can approximate the solid angle with:

$$\Omega \simeq \pi\theta^2\,, \tag{3}$$

with θ in radians of course. Notice that this is just the area of a circle of radius θ. The true solid angle will be slightly smaller than this for a given value of θ, although this is almost always appropriate for astronomical measurements. The true formula is

$$\Omega = 2\pi(1 - \cos\theta) \tag{4}$$

Intensity

The *specific intensity* of radiation the most basic observable quantity. It is essentially the surface brightness, and is appropriate for all resolved objects. By "resolved" I mean that we can in principle sense differences across the source. Stars (except for the Sun) are with very few exceptions unresolved; they are just single points of light.

Intensity is independent of distance. Thus, the camera exposure time and aperture setting for an exposure of the Sun would be the same, regardless of whether the photograph was taken close to the Sun (from near Venus, for example) or far away from the Sun (from near Mars, for example), so long *as the Sun is resolved* in the photograph. This seems terribly wrong at first, but can easily be proven.

Intensity is the same at the source and at the detector. Thus you can think of brightness in terms of energy flowing out of the source or as energy flowing into the detector.

Intensity is related to the energy dE passing through an infinitesimally small area dA by:

$$dE = I_\nu \, dA \, \cos\theta \, d\Omega \, d\nu \, dt. \tag{5}$$

Here, "specific" refers to the fact that it is at a particular wavelength. We can of course rewrite this as:

$$I_\nu = \frac{dE}{dA \cos\theta d\Omega d\nu dt}. \tag{6}$$

In this expression, θ is measured normal to the surface dA and $d\Omega$ is the solid angle. The dimensions of I_ν are then $\mathrm{erg\,cm^{-2}\,Hz^{-1}\,s^{-1}\,sr^{-1}}$. In the raio, "sr" is often replaced with "beam."

Notice that we wrote the specific intensity in frequency units. I_ν has a dependence on $d\nu$, and $d\nu \neq d\lambda$. Instead,

$$d\nu = -(c/\lambda^2) d\lambda. \tag{7}$$

So combining with the above equations, we see that

$$\nu I_\nu = \lambda I_\lambda. \tag{8}$$

To get the *intensity* or *integrated intensity* we would of course integrate over frequency or wavelength so that

$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda. \tag{9}$$

Flux

While intensity is a natural unit for extended sources, we are frequently more interested in the quantity of *flux* integrated over solid angle:

$$F_\nu = \int I_\nu \cos\theta d\Omega \tag{10}$$

or

$$F_\nu = \int_0^{2\pi} \int_0^{\pi} I_\nu \cos\theta \sin\theta d\theta d\phi \,. \tag{11}$$

This is technically the *flux density*, where "density" refers to the fact that it is at a particular wavelength or frequency. Unfortunately, flux and flux density are often used incorrectly. The units of flux density are therefore $\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$. Similar to the intensity, we can integrate over frequency or wavelength to get the *flux* or *integrated flux*.

In practice, when do we use spectral brightness and when do we use flux density to describe a source? If a source is unresolved, meaning that it is much smaller in angular size than the point-source response of the eye or telescope observing it, its flux density can be measured but its spectral brightness cannot. If a source is much larger than the point-source response, its spectral brightness at any position on the source can be measured directly, but its flux density must be calculated by integrating the observed spectral brightnesses over the source solid angle.

Luminosity

Intensity and flux are observable quantities, and not physical quantities. We are often more interested in luminosity, which is intrinsic to the source. The *observed* luminosity is

$$L = 4\pi d^2 F \,, \tag{12}$$

where d is the distance to the source.

If the object is a blackbody (and spherical), we can use Stephan-Boltzmann:

$$L = 4\pi R^2 \sigma T^4 \,, \tag{13}$$

(more on this later).

Distance

Distances to astronomical objects are a whole course of study in themselves! One such account can be found in "Measuring the Universe" by Webb (Springer/Praxis).

In any case, the fundamentals that we need to know are that much of what we know about distances are based on the parallax technique applied to the nearby stars. Parallax measures the relative shift in position of a foreground star, relative to background stars, as the Earth revolves around the Sun (or, alternatively, as a satellite revolves around the earth).

$$d = \frac{1\,\mathrm{AU}}{\tan p} \simeq \frac{1}{p}\,\mathrm{AU} \,, \tag{14}$$

where d is the distance and p is the measured parallax. This is so useful, that we usually use "arcseconds," 1/3600 of a degree, to measure p, and then the distance d can be measured in pc:

$$d = \frac{1}{p}\,\mathrm{pc} \,. \tag{15}$$

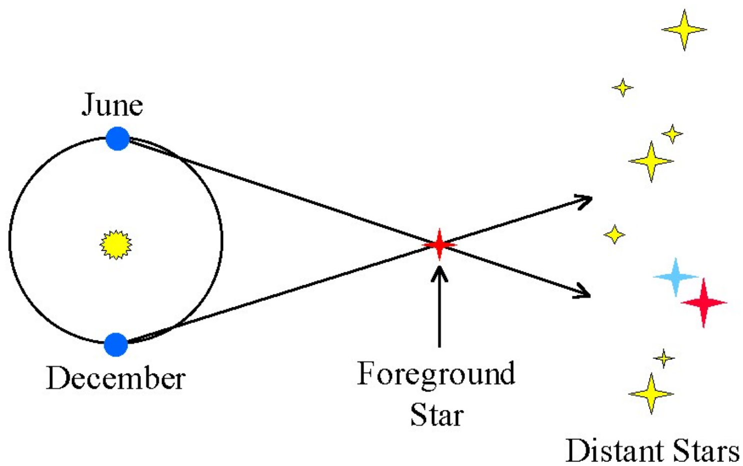

Figure 1: Parallax

(Apparent) Magnitude

Magnitudes are the units of brightness, typically used in the optical and near-infrared regimes. They are always measured in a particular bandpass, for example the Johnson V-band. This allows us to compute "colors" by looking at magnitude differences. Colors are the crudest way of determining the shape of the spectral energy distribution.

Magnitudes are based on Hipparchus's classification of stars in the northern sky. Hipparchus classified stars with values of magnitudes from 1 to 6, 1 magnitude being the brightest. Because it was defined by eye, and the eye does not have a linear response, a first magnitude star is not twice as bright as a second magnitude star. Instead, astronomers later found that Hipparchus' system is roughly logarithmic, and 6th magnitude stars are roughly 100 times fainter than 1st magnitude stars. The magnitude system has two peculiarities:
(1) It is defined backwards, and
(2) It is logarithmic.
So it is basically the ideal system to use.

Five equal steps in log-space (1st to 6th magnitude) result in factors of 2.512 in linear space ($100^{\Delta m/5} = 2.512^{\Delta m}$). Therefore, a 1st magnitude star is 2.512 times brighter than a second magnitude star, and a 4th magnitude star is $2.512^3 = 15.8$ times fainter than a 1st magnitude star. Another way of thinking about this is:

$$m_1 - m_0 = -2.5 \log_{10}(F_1/F_0) \tag{16}$$

or

$$\frac{F_1}{F_0} = 10^{0.4(m_0 - m_1)}\,, \tag{17}$$

where F_1 and F_0 are the fluxes and m_0 and m_1 are the magnitudes of stars "0" and "1".

The magnitude of a star is usually measured at a particular wavelength, through a photometric filter. The most common filters used are the Johnson U, V, B, R, I, but there are now a large number of filters available. Magnitudes found using these filters are often denoted with the filter names themselves, e.g., B for m_B.

How can we actually use this system? We need a reference star of known flux (F_0) and magnitude (m_0). Any star will do, but two commonly used ones are Vega and the Sun. Our book lists magnitudes of the Sun in common filters, in Appendix A.

If we wish to convert from the magnitude measured in one filter to that over all wavelengths (the "bolometric" magnitude), we need a "bolometric correction," BC.

$$m_{\rm bol} = m_V + BC \tag{18}$$

For the Sun, $\mathrm{BC}_\odot = -0.08$.

Many things in space attenuate (absorb/scatter) star light, and this attenuation is often measured in magnitudes. For example, each kpc in the Galaxy produces about a magnitude of visual extinction. Star formation regions can have visual extinctions of 100, so a star would have $2.5^{100} = 10^{40}$ times less light than it would if extinction were not present. Extinction generally decreases with increasing wavelength, so it is less in the infrared and essentially absent in the radio.

Absolute Magnitudes

We can also use absolute magnitudes to denote a quantity similar to luminosity, i.e., something intrinsic to the source. In this case, instead of flux, we simply replace with luminosity.

$$M_1 - M_0 = -2.5 \log_{10}(L_1/L_0) \tag{19}$$

or

$$\frac{L_1}{L_0} = 10^{0.4(M_0 - M_1)}\,, \tag{20}$$

Note, capital M refers to absolute magnitude, lower case is apparent.

But how exactly is the absolute magnitude defined? It is the apparent magnitude of a star at a distance of 10 pc of course! So

$$m - M = 5 \log d - 5\,, \tag{21}$$

where d is in pc. If we plug 10 in for d, we find

$$m - M = 5 \times 1 - 5 = 0\,, \tag{22}$$

so $m = M$.

The quantity $\mu = m - M$ is known as the "distance modulus," and uses the symbol μ.

Astronomical filters

We cannot observe *all* the light from an object, that's just not how detectors (including your eyes) work. Instead, we use filters that have a certain response shape, a certain bandpass, and a certain central frequency.

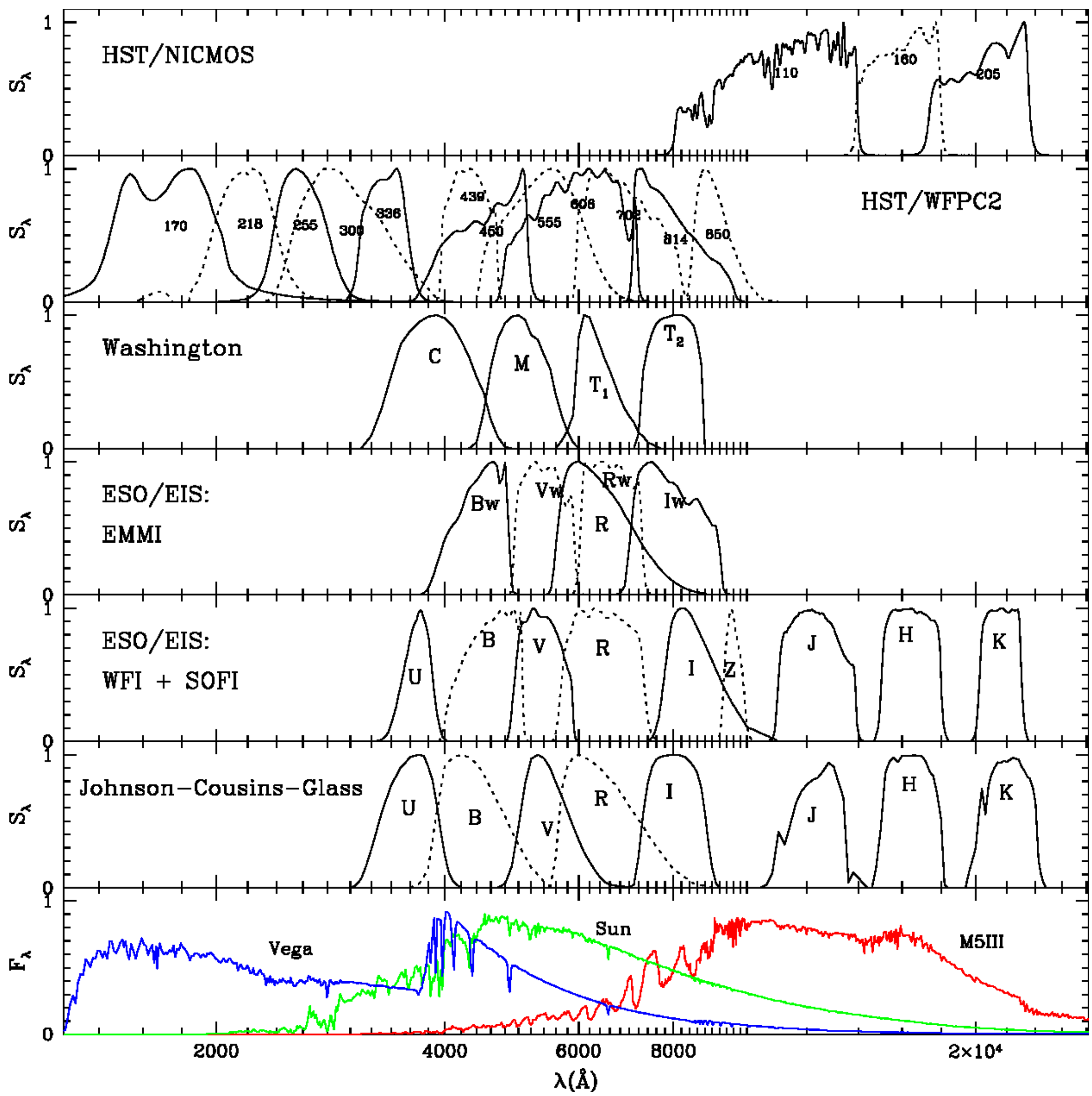

Figure 2: Common astronomical filters.

Colors

Differences in magnitudes are (confusingly) called colors. A color gives the relative intensity between two wavelengths, similar to a slope. We'll chat about colors more later.

In-Class questions

1) Tycho Brahe looked for stellar parallax, but couldn't detect it due to the precision of his instruments. If the nearest star, Proxima Certauri, is 1.3 pc from the Sun, what can you say about Tycho's angular resolution?
2) By what factor is the flux of a 5th magnitude star less than that of a 3rd magnitude star?
3) If the absolute V-band magnitude of the Sun is +4.8, what is its apparent V-band magnitude?
4) The Sun has an absolute V-band magnitude of +4.8 and an absolute B-band magnitude of +5.5. What is its flux ratio at the two wavelength filters?