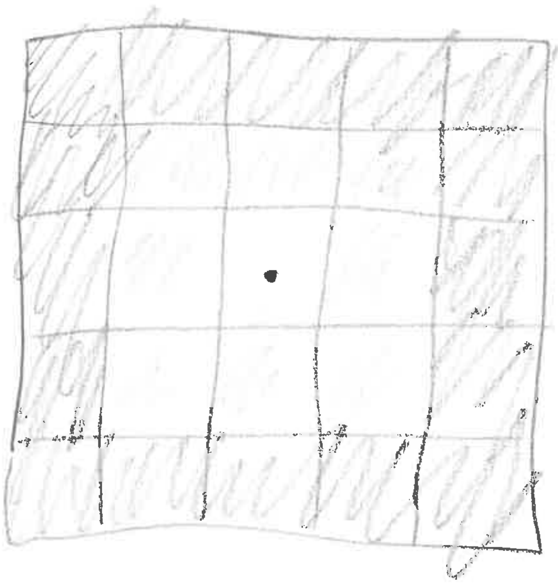
ASTR 368

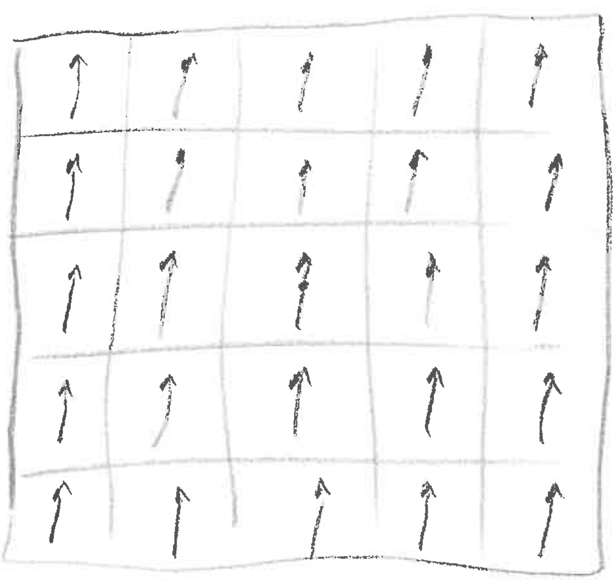
HW #6

1) Isotropic, not homogeneous



Shading maybe indicates galaxy # density

Homogeneous, not isotropic



Arrows indicate velocity

2) Collapse!

3) a) $H^2[1-\Omega]R^2 = -kc^2$

$$H_0^2[1-\Omega_0] = -kc^2$$

b) $\left(\frac{1}{R}\frac{dR}{dt}\right)^2\left[1-\frac{\rho}{\rho_c}\right]R^2 = -kc^2$

$$\rho = \frac{\rho_0}{R^3} \qquad \rho_c = \frac{3H^2}{8\pi G} = \frac{3(dR/dt)^2}{8\pi G R^2}$$

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2\left[1-\frac{\rho_0}{R^3}\cdot\frac{8\pi G \cancel{R^2}}{3\,(dR/dt)^2}\right] = -kc^2$$

$$\left[\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_0}{3R^3}\right] = -kc^2$$

c) If $\rho = \rho_c$ and $k=0$,

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho_0}{3R}$$

or

$$\int_0^R R^{1/2}\,dR = \left(\frac{8\pi G\rho_0}{3}\right)^{1/2}\int_0^t dt$$

$$\frac{2R^{3/2}}{3} = \left(\frac{8\pi G\rho_0}{3}\right)^{1/2} t$$

$$\Rightarrow R = 3^{2/3}(8\pi G\rho_0)^{1/3}\,3^{-1/3}\,t^{2/3} = (6\pi G\rho_{c,0})^{1/3}\,t^{2/3}$$

d) $\rho_{c,0} = \frac{3H_0^2}{8\pi G}$ $t_H = \frac{1}{H_0}$

$$R = (6\pi G \rho_{c,0})^{1/3} t^{2/3}$$

$$= \left(6\pi G \cdot \frac{3H_0^2}{8\pi G}\right)^{1/3} t^{2/3}$$

$$= \left(\frac{18}{8t_H^2}\right)^{1/3} t^{2/3} = \left(\frac{9}{4}\right)^{1/3} \left(\frac{t}{t_H}\right)^{2/3}$$

c) Age of Universe is when $R = 1$

$$1 = \left(\frac{9}{4}\right)^{1/3} \left(\frac{t}{t_H}\right)^{2/3}$$

$$1 = \frac{9}{4} \left(\frac{t}{t_H}\right)^2$$

$$t = \left(\frac{4}{9}\right)^{1/2} t_H = \frac{2}{3} t_H$$

4) $H^2[1-\Omega]R^2 = -kc^2$

$H^2[1-(\Omega_m + \Omega_{rad})] = -kc^2$

It would reduce the expansion.

$P = w\rho c^2$

for matter, $w=0$

Since $\rho \propto R^{-3(1+w)}$, $\rho \propto R^{-3}$

for radiation, $w = +1/3$, so $\rho \propto R^{-4}$

(for DE, $w \approx -1$, so $\rho \propto R^0$)