ISM HW #3

1) a) Flat $v(R) = \text{const}$

$$\omega(R) \propto \frac{v}{R} \propto \frac{1}{R}$$

$$\frac{v^2}{R} = \frac{G M_{int}}{R^2} \Rightarrow M_{int} = \frac{v^2 R}{G} \propto R$$

$$\rho \propto M_{int} / R^3 \propto R^{-2}$$ (not quite, 9...)

b) Keplerian $P^2 \propto a^3$

$$v(R) \propto \frac{a}{P} \propto \frac{a}{a^{3/2}} \propto R^{-1/2}$$

$$\omega(R) \propto \frac{v}{R} \propto R^{-3/2}$$

$$M_{int} = \frac{v^2 R}{G} \propto R^{-1} R \propto \text{const}$$ [all mass at center]

$$\rho \propto M_{int} / R^3 \propto R^{-3}$$

c) Solid body $v(R) \propto R$

$$\omega(R) \propto \frac{v}{R} \propto \text{const}$$

$$M_{int} = \frac{v^2 R}{G} \propto R^3$$

$$\rho \propto M_{int}/R^3 \propto \text{const}$$

2a) In the limit that $\tau >> 1$, $T_B \to T_s$. For this galaxy, its rotation spreads the HI emission over ~200 km/s. The left peak is therefore from the blue-shifted side of the galaxy; right peak from redshifted side.

If $\tau >> 1$, we would see the same intensity at all velocities.

b) Peaks are ~175 km/s apart, so ±85 km/s about 900 km/s.

Equate (circular) centripital accel w/ gravitational accel.

$$\frac{v^2}{R} = \frac{G M_{int}}{R^2} \Rightarrow M_{int} = \frac{v^2 R}{G}$$

Taking $v = 85$ km/s, $M_{int} = 44 \times 10^{10} M_\odot$

This is total mass, not just HI

c) Area under the curve is ~200 km/s · 0.3 Jy

$\simeq 60$ Jy km/s

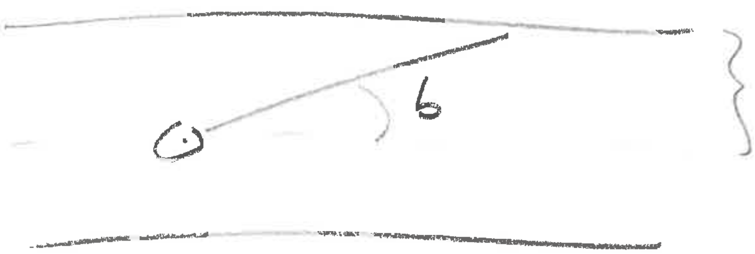
$$d = \frac{v}{H_0} \simeq \frac{900 \text{ km/s}}{67 \text{ km/s/Mpc}} = 13.4 \text{ Mpc}$$

$$M_{HI} = 2.343 \times 10^5 M_\odot \underbrace{(1+z)}_{\approx 1} \left(\frac{13.4 \text{ Mpc}}{\text{Mpc}}\right)^2 \left(\frac{60 \text{ Jy km/s}}{\text{Jy km/s}}\right)$$

$$= 2.5 \times 10^9 M_\odot$$

d) Dark matter! And stars. And other gas.

3a)



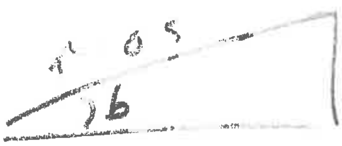
$3\times10^{20}\ \mathrm{cm}^{-2}$

$$\tau = 7.19\ \frac{N(HI)}{10^{21}\ \mathrm{cm}^{-2}}\ \frac{100}{T_s}\ \frac{\mathrm{km/s}}{\sigma_v}\ e^{-v^2/2\sigma_v^2}$$

@ line center, exponential term = 1,

$\tau = 0.0657$ for $N(HI) = 3\times10^{20}\ \mathrm{cm}^{-2}$,

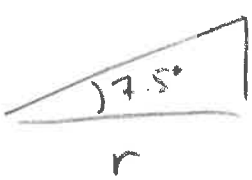
or, i.e. $b = 90^\circ$



$\tau = 0.0657$

$$\sin b = \frac{0.0657}{0.5} \Rightarrow b = 7.5^\circ\ ;\quad |b| > 7.5^\circ$$

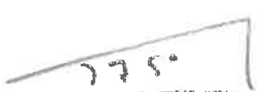
b)



$$\tan 7.5^\circ = \frac{150\,\mathrm{pc}}{r}$$

$\Rightarrow r = 1.13$ kpc

c)



$$\sin 7.5 = \frac{3\times10^{20}\ \mathrm{cm}^{-2}}{N(HI)}$$

$\Rightarrow N(HI) = 2.28\times10^{21}\ \mathrm{cm}^{-2}$

4) $g(\nu) = C e^{-(\nu-\nu_0)^2/2\sigma^2}$ (not normalized)

$$\int_{-\infty}^{\infty} g(\nu)\,d\nu = C \int_{-\infty}^{\infty} e^{-(\nu-\nu_0)^2/2\sigma^2}\,d\nu$$

Integral table

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2}\,dx = \sqrt{\frac{\pi}{a}} \quad ; \quad a = \frac{1}{2\sigma^2}$$

$$\Rightarrow \int_{-\infty}^{\infty} g(\nu)\,d\nu = C\,(2\pi\sigma^2)^{1/2} = 2.507\,C\sigma$$

FWHM $= 2.355\,\sigma$ so

$$\frac{2.507 - 2.355}{2.507} = 6.1\%$$