ASTR705 ISM
HW #8
Due Friday, 4/14 in class

1) The pulses from a pulsar arrive later at low frequencies than at high frequencies. Suppose that the arrival time at 1420 MHz and 1610 MHz differ by $\Delta t(1420\,\text{MHz},\ 1610\,\text{MHz}) = 0.0913\,\text{s}$.
a) What is the DM for this pulsar?
b) If the pulsar is at a distance of 6 kpc, what is the mean electron density along the line of sight?

2) A pulsar is observed at 1610 and 1660 MHz. The plane of polarization at these two frequencies differs by 57.5°.
a) What is the minimum possible magnitude of the RM toward this source? Why is it a minimum?
b) What would be the next largest possible value for RM?
c) If the source has a $DM = 200$ cm^{-3} pc, and using the minimum RM from part a), what is the electron density weighed component of the magnetic field along the line of sight?

3) The brightest part of the Orion H II region has an emission measure $EM = \int n_e^2 dl \simeq 5 \times 10^6\,\text{cm}^{-6}\,\text{pc}$. The FWHM line width of the H90α ($n = 91 \rightarrow 90$) RRL is 25 km s^{-1}. Assume equal parts thermal and turbulent broadening.
a) What is the optical depth τ due to free-free absorption at $\lambda = 1\,\text{cm}$ ($\nu = 30\,\text{GHz}$)? Use κ_{ff}.
b) What is the optical depth τ due to free-free absorption at $\lambda = 21\,\text{cm}$ ($\nu = 1420\,\text{MHz}$)?
c) The main Orion H II region is $\sim 2.5'$ in diameter, and 500 pc from the Sun. What is the mean electron density in the brightest part of the nebula?

4) Calculate the optical depth at line center for H$n\alpha$ radiation propagating through a slab of pure hydrogen. Assume that $n \gg 1$, so radio emission. You can use the approximate frequency given in Draine Chapter 10, as well as the approximate Einstein As from the same chapter. You'll want to use

$$\tau = \sigma_{\ell u} \left(1 - \frac{n_u}{n_\ell}\frac{g_\ell}{g_u}\right) N_n \qquad (1)$$

and use Equation 3.45 for N_n. Leave your answer in terms of the principle quantum number n, EM, T, and σ, as well as the departure coefficient b_n and the factor β_n (we didn't get to these in class; see notes). b) Assume $EM = 10\,\text{cm}^{-6}\,\text{pc}$ for a 1 pc thick slab, $b_n = 0.9$ and $\beta_n = -100$. Evaluating your answer for a), what is the optical depth for the 166α transition of hydrogen? What is the frequency of this transition?