ASTR 367

HW #7

1) $MV = \text{const}$

$$\frac{M_{sun}}{M} = \frac{V}{V_{sun}}$$

$$V = \frac{4}{3}\pi R^3$$

$$\Rightarrow \left(\frac{R_\odot^3 \, M_\odot}{0.5 M_\odot}\right)^{1/3} = R$$

$$\Rightarrow R = R_\odot \cdot 0.5^{-1/3} = 1.26 R_\odot$$

2) 1.4 $M_\odot$ is the maximum mass before
$P_g > P_{degeneracy}$. If we add 50% more
mass $\frac{1.5}{1} = \left(\frac{R_{old}}{R}\right)^3 \Rightarrow R = 1.5^{1/3} R_{old} = 1.15 R_{old}$

$$a_{rot} = \frac{v^2}{R} = \omega^2 R$$

$$a_g = \frac{GM}{R^2}$$

$$P_{degeneracy,old} = P_{g,old}$$

$$P_{degeneracy,new} = P_{g,new} - P_{rot}$$

Since $P \propto F \propto a$, can work w/ acceleration

$$a_{g,old} = a_{g,new} + a_{rot}$$

$$\frac{GM_{old}}{R_{old}^2} = \frac{GM_{new}}{R_{new}^2} - \omega^2 R_{new}$$

$$\omega = \left[\frac{GM_{old}}{R_{new}}\left(\frac{1}{R_{old}^2} + \frac{M_{new}/M_{old}}{R_{new}^2}\right)\right]^{1/2} = \left[\frac{GM_{old}}{R_{new}^3}\left(\left(\frac{R_{new}}{R_{old}}\right)^2 + 1.5\right)\right]^{1/2}$$

$$= \left[\frac{GM_{old}}{R_{new}^3} \cdot 1.98\right]^{1/2}$$

3) $\Delta P \cong 10^{-8} P$

a) $R = \left(\frac{P^2 GM}{4\pi^2}\right)^{1/3}$

$dR = \left(\frac{GM}{4\pi^2}\right)^{1/3} \frac{2}{3} P^{-1/3} dP = \frac{2\times10^{-8}}{3}\left(\frac{GM}{4\pi^2}\right)^{1/3} P^{2/3}$

$M = 2.8\times10^{30}\,kg$ $\quad\quad P = 0.033\,s$

$\Rightarrow dR = 0.0011\,m$

b) $\frac{\omega_f}{\omega_i} = \left(\frac{R_i}{R_f}\right)^2 = \left(\frac{5\times10^6 m}{10^4 m}\right)^2 = 2.5\times10^6$ (approx, for core)

Sun rotates in about 27^d, so

$27^d / 2.5\times10^6 = 1.08\times10^{-5}\,d = 0.93\,s$

c) $B_f = B_i \cdot 2.5\times10^6$

$B_i \sim 1G$, so $B_f \approx 2.5\times10^6\,G = 250T$

d) If the radius is unchanged, the rotate speed would stay the same

4) Assume $\rho_{WD} = \frac{M_\odot}{4/3\pi R_\oplus^3} = 1.83\times10^9\ kg/m^3$

$V_{tsp} = 5\ cm^3 = 5\times10^{-6}\ m^3$

$m_{elephant} \cong 5000\ kg$

Mass of 1 tsp WD: $\rho \cdot V_{tsp} \cong 10^4\ kg$

a) $\frac{10^4\ kg}{5\times10^3\ kg} \sim 2$ elephants

b) $\rho_{ns} \sim \frac{M_\odot}{4/3\pi\ (10\times10^3)^3} \cong 5\times10^{17}\ kg\ m^{-3}$

Mass of 1 tsp of NS: $\rho V_{tsp} \cong 2.5\times10^{12}\ kg$

$\frac{2.5\times10^{12}}{5\times10^3} = 5\times10^8$ elephants

c) Assume $1\ M_\odot = 2\times10^{30}\ kg$

$\frac{2\times10^{30}}{5\times10^3} = 4\times10^{26}$ elephants