ASTR 367

HW #10

1) $\theta_\odot = \dfrac{7\times10^8\ m}{1.5\times10^{11}\ m} = 0.0047\ rad$

$\theta_{mercury} = \dfrac{2.4\times10^6\ m}{9.0\times10^{10}\ m} = 0.000027$

0.6 AU distance varies in it's elliptical orbit

$\Omega = \pi\theta^2$ so

$$\frac{\Omega_\odot - (\Omega_\odot - \Omega_{mercury})}{\Omega_\odot} = \left(\frac{\theta_{mercury}}{\theta_\odot}\right)^2 = 0.003\%$$

2)

$$\frac{a_1}{a_2} = \frac{m_2}{m_1}$$

$$a_1 = a_2 \frac{m_2}{m_1}$$

$$a_2 = a_1 \frac{m_1}{m_2}$$

$$a = a_1 + a_2 = a_2 \frac{m_2}{m_1} + a_2 = a_2\left(\frac{m_2}{m_1} + 1\right) = a_1 \frac{m_1}{m_2}\left(\frac{m_2}{m_1}+1\right)$$

$$= a_1\left(1 + \frac{m_1}{m_2}\right)$$

~~a/a~~ $$\frac{a_2}{a_1} = \frac{1 + \frac{m_1}{m_2}}{\frac{m_2}{m_1}+1} = \frac{\frac{m_2+m_1}{m_2}}{\frac{m_2+m_1}{m_1}} = \frac{m_1}{m_2}$$

$\vec{r} = \vec{r}_2 - \vec{r}_1$ but $\frac{r_2}{r_1} = \frac{a_2}{a_1}$ so

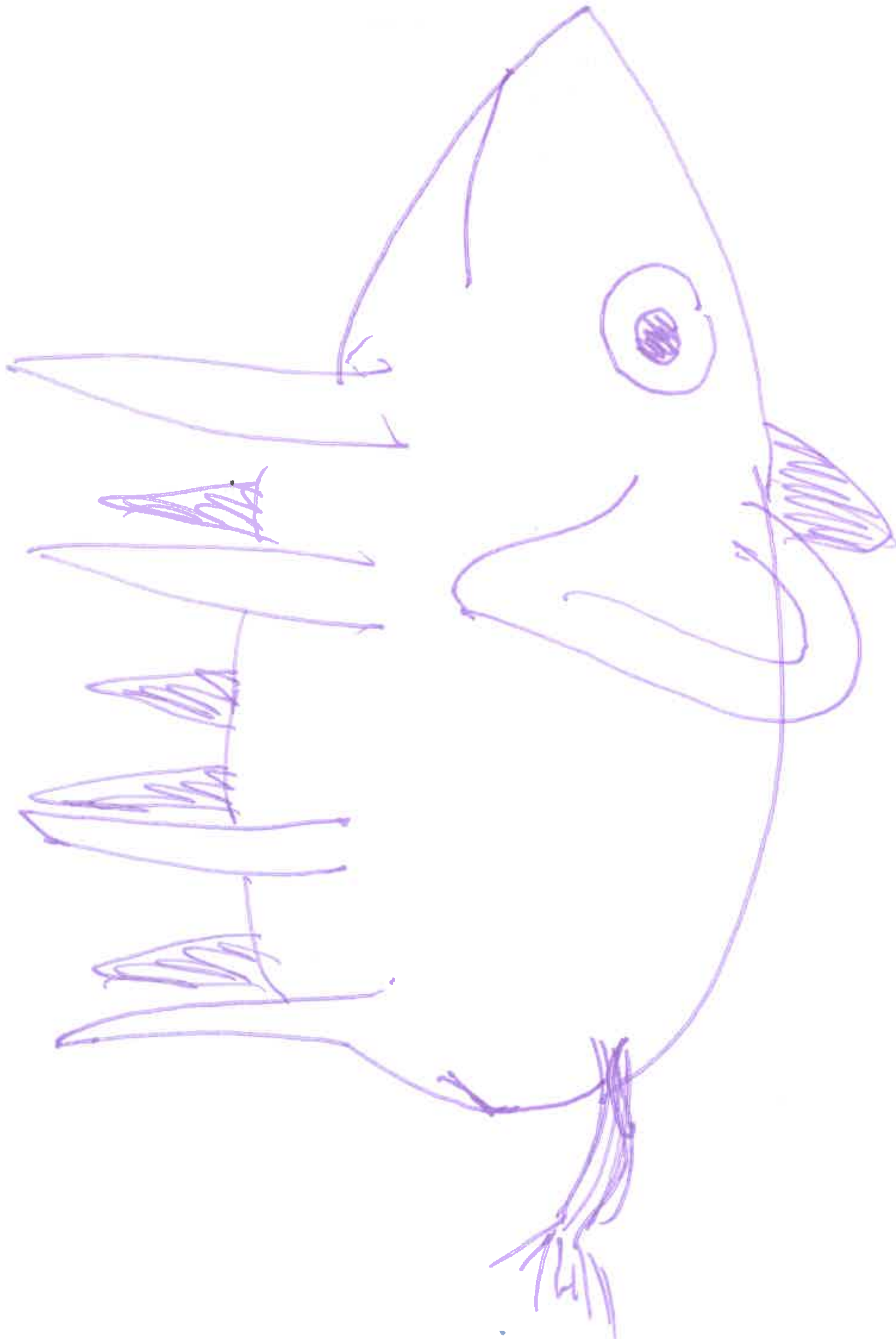
$$r = r_2\left(\frac{a_1+a_2}{a_2}\right)$$

$a = \frac{1}{2}(r_p + r_a)$

$$r = (a_1+a_2)\left(\frac{1-e^2}{1+e\cos\theta}\right)$$

$= \frac{1}{2}\left[(a_1+a_2)(1-e) + (a_1+a_2)(1+e)\right]$

$= a_1 + a_2$



3) $P = 50$ yr

a) $p = \frac{1}{d} \Rightarrow d = 2.5$ pc

b) $a = 7.6'' = 2.8 \times 10^{12}$ m   e $d = 2.5$ pc

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$$m_1 + m_2 = \frac{4\pi^2 a^3}{GP^2} = \frac{4\pi^2 \cdot (7.8 \times 10^{12} m)^3}{G \cdot (50 yr)^2}$$

$$= 5.7 \times 10^{30} \text{ kg}$$

c) If v is nonzero,

d) $m_v = -1$ $m - M = 5 \log d - 5$

$\Rightarrow M = 2.0$

$$\frac{L}{L_\odot} = 10^{0.4(+4.8-2.0)} = 13$$

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} \Rightarrow \frac{M}{M_\odot} = 2.1$$

mass function

$$\frac{m_2^3}{(m_1+m_2)^2}\sin^3 i = \frac{P}{2\pi G} v_{1r}^3$$

If $m_1 + m_2 = 2.5$ and $m_1 = 2.0$,

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{0.5^3}{2.5^2} = \frac{0.125}{6.25} = 0.02$$

4) $\frac{m_1}{m_2} = \frac{a_2}{a_1}$

$m_{jup} = 1.9 \times 10^{27}$ kg

$m_{\odot} = 2.0 \times 10^{30}$ kg

$a_{jup} = 7.78 \times 10^{11}$ m

Assume $a = a_{jup} + a_{\odot} \approx a_{jup}$

$a_{\odot} = a_{jup} \cdot 0.00095 = 7.39 \times 10^{8}$ m

$$v = \frac{2\pi a}{P} = \frac{2\pi a}{\left(4\pi^2 / G(M_{\odot} + M_{jup}) a^3\right)^{1/2}}$$

Assume $M_{\odot} + M_{jup} = M_{\odot}$

$v = \left(\frac{GM_{\odot}}{a}\right)^{1/2} = 425$ ~~km~~/s fast!

should be ~13 km/s