ASTR 702
HW #6

1) $L_{edd} = \frac{4\pi G M c}{\kappa}$

If $\kappa = \kappa_{es}$, we can assume $\kappa = \frac{\sigma_T}{m_p}$

$\Rightarrow L_{edd} = \frac{4\pi G M m_p c}{\sigma_T}$

$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha$

$L_\odot \left(\frac{M}{M_\odot}\right)^\alpha = \frac{4\pi G M m_p c}{\sigma_T}$

$\Rightarrow M = \left(\frac{4\pi G m_p c \; M_\odot^\alpha}{L_\odot \sigma_T}\right)^{\frac{1}{\alpha - 1}}$

If $\alpha = 3.5$, $M = 65 M_\odot$

If $\alpha = 3.0$, $M = 180 M_\odot$

2) We know $P \propto F \propto a$, so the Chandrasekhar limit can be cast in terms of acceleration.

$a_{deg} = a_g$ in the limit

$a_{deg,new} = a_{g,new} - a_{rot}$

but

$a_{deg,old} = a_{g,old}$

so

$a_{g,old} = a_{g,new} - a_{rot}$

$$\frac{GM_{old}}{R_{old}^2} = \frac{GM_{new}}{R_{new}^2} - w^2 R_{new}$$

$$w = \left(\frac{GM_{new}}{R_{new}^3} - \frac{GM_{old}}{R_{old}^2 R_{new}} \right)^{1/2}$$

For WDs, $\frac{M_{old}}{M_{new}} = \left(\frac{R_{new}}{R_{old}} \right)^3$

3) Lane-Emden is 2^{nd} order, but we need 1^{st} order to use Euler's method

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n$$

$$\frac{1}{\xi^2}\left[2\xi\frac{d\theta}{d\xi} + \xi^2\frac{d^2\theta}{d\xi^2}\right] = -\theta^n$$

$$\frac{2}{\xi}\frac{d\theta}{d\xi} + \frac{d^2\theta}{d\xi^2} = -\theta^n$$

If $f(\xi) = \frac{d\theta}{d\xi}$,

$$\frac{df}{d\xi} = -\theta^n - \frac{2f}{\xi}$$

We can now do Euler's method with

$$\theta(\xi_i+1) = -\theta^n + \frac{2}{\xi}f(\xi_i)$$

$$f(\xi_i+1) = f(\xi_i) + h\left[-\theta^n(\xi_i) - \frac{2}{\xi_i}f(\xi_i)\right]$$

with step size h

So density $\rho = \rho_c\theta^n$ and $0 > \theta > 1$