

# Stellar Structure

## Stars

### What is a Star?

A star is any object that satisfies two conditions:

1. bound by self-gravity (spherically symmetric)
2. radiates energy supplied by an internal source (nuclear fusion or gravitational potential energy)

Because energy in #2 above is limited, stars evolve and “die.”

Stars are remarkably simple objects. We can almost fully characterize them with only their masses! The Russell–Vogt theorem states that, if we know a star’s mass and its chemical composition, we can determine all of its other properties. We will spend time talking about the following features:

1. Mass
2. Spectral Type
3. Luminosity
4. Radius
5. Temperature
6. Composition

### Mass

Stellar masses range from  $0.08 M_{\odot}$  to  $\sim 100 M_{\odot}$ . Below  $0.08 M_{\odot}$ , gravitational pressure is insufficient to sustain fusion (more on this later). Above  $\sim 100 M_{\odot}$ , the star’s fusion is so powerful that gravity is not able to hold it together.

Massive stars are rare, and low mass stars are common. The stellar birth rate is given by the “initial mass function,” or IMF.

We can see from the IMF that the most common mass less than a Solar mass and the numbers decrease at higher masses. Note that this is the *initial* mass! To get the present-day mass function, we have to account for stellar lifetimes, and as we’ll see, low-mass stars live for a long time.

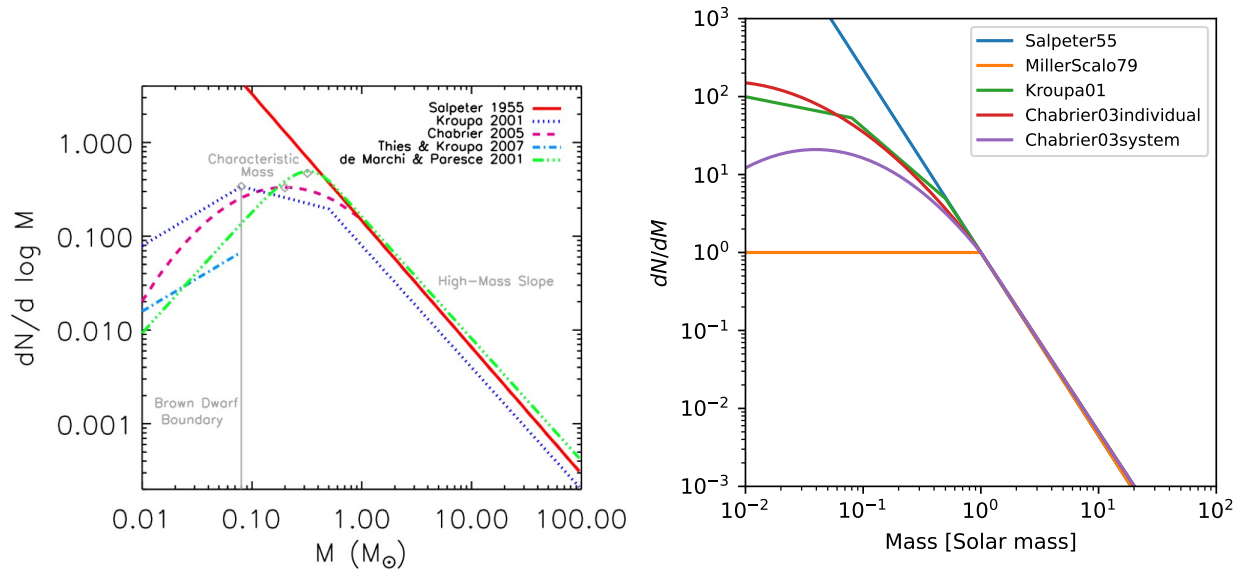


Figure 1: The initial mass function as derived by various authors. (left) Please note the y-axis, which is the number of stars per *logarithmic* mass bin. The Chabrier (2005) IMF is log-normal. (right) The same, but for linear mass bins.

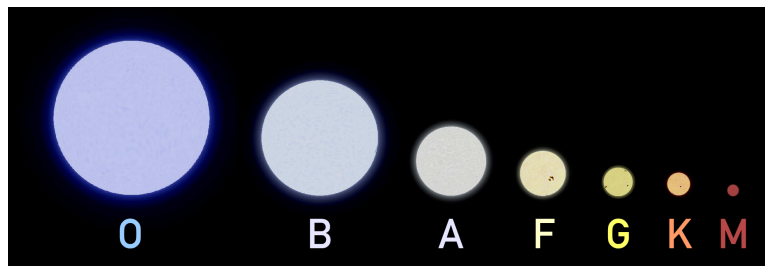


Figure 2: Relative sizes of spectral types.

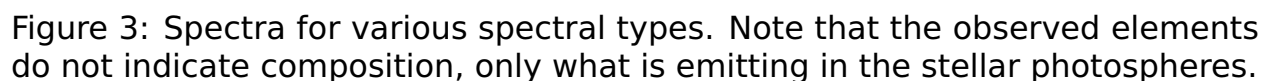
## Spectral Type

[Note: this is just a proxy for mass in most cases]

Astronomers classify stars based on their spectra, or energy emitted as a function of frequency. Stellar spectra show absorption lines, and stars were initially classified based on the strength of the hydrogen absorption lines, from type “A” down through the alphabet.

Later, astronomers realized that a better classification scheme would rank stars in order of their temperature (or mass or luminosity). Thus, we have the spectral types OBAFGKM ranging from the largest stars (O) to the smallest (M). A common mnemonic is “Oh, be a fine girl/guy, kiss me!”, which sounds quaint, in the inappropriate way of old sayings.

Within these, we subdivide into numbers, with 0 the largest, followed by 1 and



For shorthand, we call large stars “early type” and small stars “late type.” We can also refer to stars as being earlier or later than a given classification.

Stellar spectra can be really complicated, and there are a number of other characters that can follow the spectral type designation, to denote various other features of the spectra. These are too numerous and obscure to focus on here.

## Luminosity

Luminosity is energy emitted per second, measured in W or erg/s. Because we measure luminosity at a particular wavelength or frequency, we often use

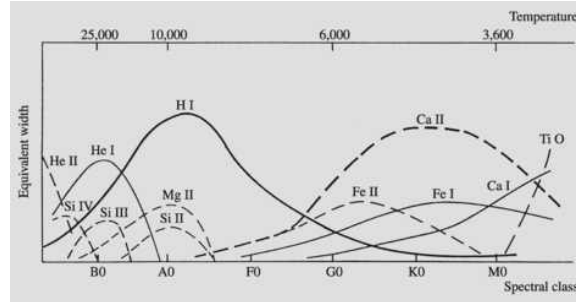


Figure 4: Strengths of various elemental and molecular absorption lines.

“spectral luminosity,” which has units of  $W/(\text{wavelength or Hz})$  or  $\text{erg/s}/(\text{wavelength or Hz})$ . Stellar luminosities range from  $10^{-5} L_{\odot}$  to  $10^5 L_{\odot}$ . The Solar luminosity is  $4 \times 10^{33} \text{ erg s}^{-1}$ .

Empirically, we have found the “mass-luminosity” relationship:

$$L = L_{\odot} \left( \frac{M}{M_{\odot}} \right)^{\alpha} \quad (1)$$

This relationship arises because more massive stars are more efficient at producing energy, and so have way more luminosity. We will discuss this further in the lectures on fusion.

The power-law index  $\alpha$  varies from about 3–5, depending on the mass. A good average value is maybe  $\alpha = 4$ .

## Radius

[Note: this is just a proxy for mass in most cases]

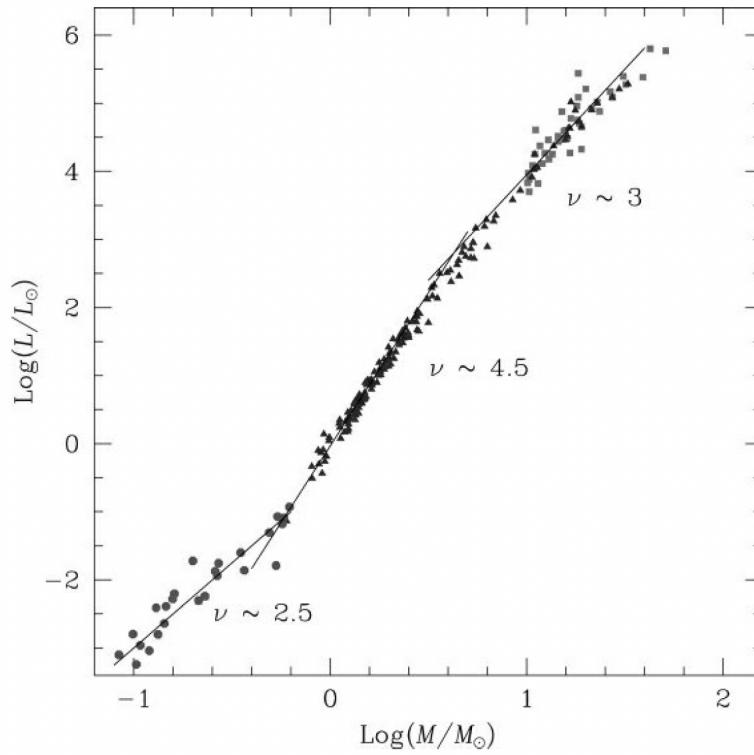
Stellar radii range from  $0.01 R_{\odot}$  to  $100 R_{\odot}$ . Stellar mass is the fundamental quantity, not radius, and this range is simply a reflection of the radii of stars having the specified mass range.

## Chemical Composition (Metallicity)

In astronomy, we define “metallicity” in a strange way, with all elements more massive than helium being called “metals.” Furthermore, we denote  $X$  the hydrogen fraction,  $Y$  the helium fraction, and  $Z$  the metals fraction, and therefore  $X + Y + Z = 1.00$ .

For the Sun,  $X_{\odot} = 0.74$ ,  $Y_{\odot} = 0.25$ , and  $Z_{\odot} = 0.0134$ .

Metallicity is generally measured as the abundance of a particular element relative to hydrogen, most commonly Iron (Fe). Iron is among the easiest to



**Figure 1.6** The mass-luminosity relation for main-sequence stars. Data from O. Yu. Malkov (2007), *Mon. Not. Roy. Astron. Soc.*, 382, based on detached main-sequence eclipsing binaries (triangles), E. A. Vitrichenko, D. K. Nadyozhin and T. L. Razinkova (2007), *Astron. Lett.*, 33 (squares) and from the compilation by O. Yu. Malkov, A. E. Piskunov and D. A. Shpil'kina (1997), *Astron. Astrophys.*, 320 (dots).

measure with spectral observations in the visible spectrum. The abundance ratio is defined as the logarithm of the ratio of a star's iron abundance compared to that of the Sun and is expressed thus:

$$[\text{Fe}/\text{H}] = \log_{10} \left( \frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\text{star}} - \log_{10} \left( \frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\odot} \quad (2)$$

where  $\frac{N_{\text{Fe}}}{N_{\text{H}}}$  is the number of iron and hydrogen atoms per unit of volume. The  $[\text{Fe}/\text{H}]$  within square brackets denotes the entire quantity in Equation 2, which can be confusing at first.

The unit often used for metallicity is the dex, contraction of “decimal exponent.” By this formulation, stars with a higher metallicity than the Sun have a positive logarithmic value, whereas those with a lower metallicity than the Sun have a negative value. For example, stars with a  $[\text{Fe}/\text{H}]$  value of +1 have 10 times the metallicity of the Sun; conversely, those with a  $[\text{Fe}/\text{H}]$  value of −1 have 1/10, while those with a  $[\text{Fe}/\text{H}]$  value of 0 have the same metallicity as the Sun. Measured stars range from metallicities of a few to maybe −4.

Metals are produced during stellar evolution (more on this later), so the lowest metallicity stars have had the least processing. We can divide stellar populations into “Population III,” the first stars in the Universe with the lowest metals, “Population II,” with higher metals, and “Population I” stars that are being born today. **Therefore, metallicity is a very rough indicator of age.** While for an individual star the age-metallicity relationship can be quite poor, for a population of stars it is meaningful.

## Lifetime

Stars typically live for ~a billion years, although the most massive stars only live for a million years and the least massive live for hundreds of billions of years (i.e., much longer than the age of the universe).

A rough estimate of the Sun's lifetime can be obtained by taking the energy available to the Sun and dividing by its luminosity. Energy is created via nuclear fusion, so  $E = Mc^2$ . But it's only about 10% efficient, so  $E = 0.1Mc^2$ . The main fusion reaction converts 4H nuclei (protons) into one He nucleus. If you look at a periodic table, 0.07% of the mass is lost in the conversion, so  $E = 0.007 \times 0.1Mc^2$ . The Solar luminosity is  $L_{\odot} = 3.8 \times 10^{33}$  erg/s and the mass is  $2 \times 10^{33}$  g. Therefore,

$$E/L \simeq \frac{0.007 \times 0.12 \times 10^{33} c^2}{4 \times 10^{33}} = 0.0005 c^2 \simeq 5 \times 10^{17} \text{ s} \simeq 10^{10} \text{ years} \quad (3)$$

The Sun has a total lifetime of about  $10^{10}$  years, and the Solar system is about 5 billion years old.

We can actually determine stellar lifetimes using the mass-luminosity relation (Equation 1). I'll leave that as an exercise.

## Temperature

[Note: this is just a proxy for mass in most cases]

Stellar temperatures range from about 4000 to 60000 K. When we talk about temperature, this is usually the temperature of the “photosphere,” the layer of the star that we see. A given star of course has a range of temperatures. The one we use is basically the stellar “surface.” The Sun’s (photospheric) temperature is 5800 K.

To understand the relationship between radius, temperature, and luminosity, we can use the Stephan-Boltzmann relation for black bodies:

$$L = A\sigma T^4, \quad (4)$$

where  $\sigma$  is the Boltzmann constant and  $A$  is the emitting area. For spherical objects (i.e. stars),  $A = 4\pi R^2$ , so

$$L = 4\pi R^2 \sigma T^4. \quad (5)$$

Since stars are basically black bodies, this equation holds for stars.

## Colors

As stated previously, “colors” are relative measures of the magnitude in two different filters. The most commonly used filters are B-blue and V-visible, so the B-V color indicates the relative intensity in this range. If B-V is positive, it means  $m_B$  is larger than  $m_V$ , so the star is red (remember: magnitudes are backwards!). If B-V is negative, the star is blue. The full range is from about +1 to -0.5. The Sun has  $B-V = 0.65$ .

## H-R Diagram

Astronomers have found it convenient to plot all stars on an H-R diagram, with the x-axis of color, temperature, or spectral type, and the y-axis of luminosity or absolute magnitude. This is known as an H-R diagram for astronomers Henry and Russell. It can describe most stellar properties, as well as evolution. It’s not hyperbole to say that it is the most important diagram in astronomy.

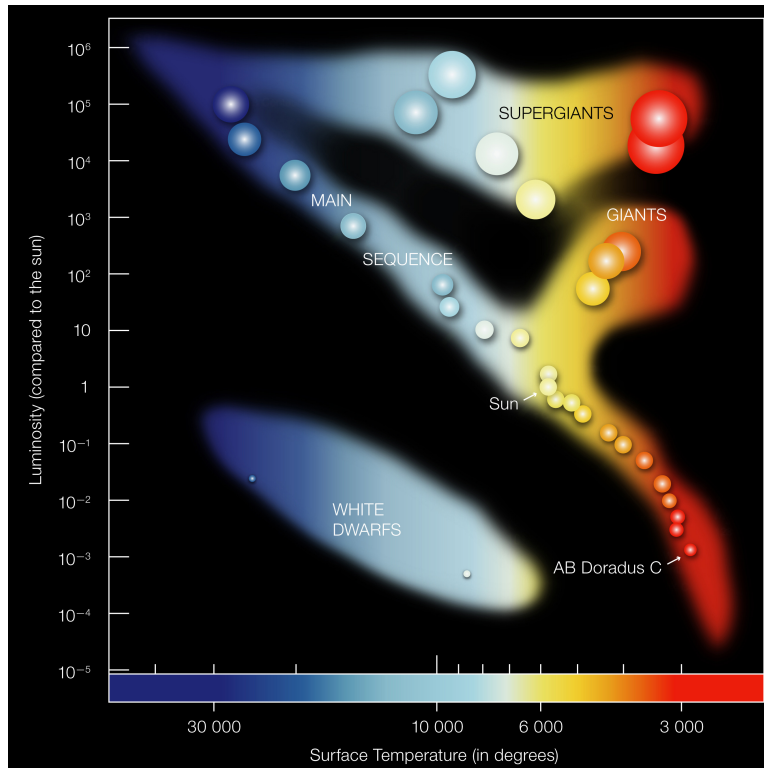


Figure 5: H-R diagram

The H-R diagram is the most important diagram in astrophysics. Running diagonally across it is the “main sequence.” There is no reason a priori to expect that there is a tight relationship between stellar temperature and stellar luminosity. The relationship is because stars are so simple. More mass leads to more luminosity (Stephan-Boltzmann).

## Vega

Vega, in the constellation Lyra, provides a very useful point of reference. Vega is an A0V star. In the night sky, Vegas is noticeably bluer than most other stars. Its B-V color is actually close to 0 though. Its temperature is about 10000 K, mass is about  $2 M_{\odot}$ .

Our other point of reference is the Sun, which is G2V, B-V= +0.65, temperature 5800 K.