ASTR 367
Interpreting Blackbody Emission

Reading: C+O, Ch 3.4

The blackbody (Planck function) is:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\,. \tag{1}$$

or

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}\,, \tag{2}$$

where the function is evaluated at frequency ν or wavelength λ, and the object is at temperature T. These functions are shown in Figure 1 for different temperatures.

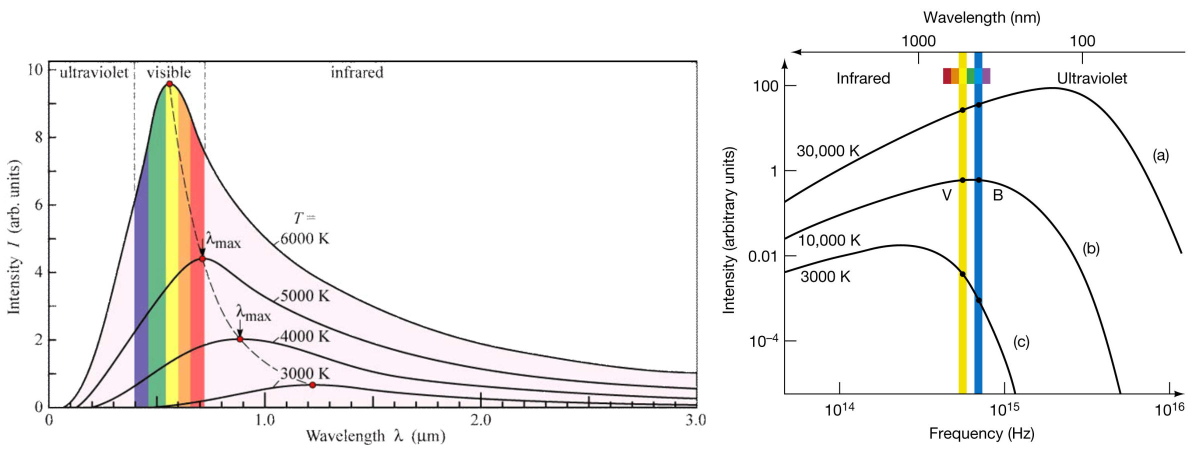

Figure 1: Blackbody curves in linear (left) and log (right)-space. Wien's Law can clearly be seen.

Students are often confused by the units: $\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}\,sr^{-1}}$ ($\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$) for B_ν or $\mathrm{erg\,cm^{-2}\,s^{-1}\,cm^{-1}\,sr^{-1}}$ ($\mathrm{W\,m^{-2}\,cm^{-1}\,sr^{-1}}$)for B_λ, where the additional "Hz" or "cm" term is the frequency or wavelength (often given in Angstroms). This also means that it is a surface brightness or an intensity.

The fundamental observational quantity in astronomy is the specific intensity I_ν. But under what conditions is $I_\nu = B_\nu$? When something called the "optical depth" τ is high. Looking ahead a little to radiative transfer,

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + B_\nu(T)(1-e^{-\tau})\,, \tag{3}$$

where $I_\nu(0)$ is the background radiation and τ_ν is the optical depth. So as $\tau \to \infty$, $I_\nu(\tau_\nu) \to 0 + B_\nu(T)(1-0) = B_\nu(T)$. To summarize, the Planck function has units of specific intensity or surface brightness, and in the limit of high optical depth, $I_\nu = B_\nu$.

There are a two important points about blackbody radiation.

1. The wavelength (or frequency) of peak intensity is inversely related to the temperature via Wien's Law:
$$\lambda_{\rm max} = \frac{0.2898}{T(\,{\rm K})}\ {\rm cm}\,, \tag{4}$$
or
$$\nu_{\rm max} = 5.879 \times 10^{10} T(\,{\rm K})\,. \tag{5}$$
We can derive these by setting the differential of B_λ or B_ν equal to zero. This tells us that hotter things peak at shorter wavelengths and higher frequencies.

   In the infrared, we have some handy rules of thumb: a 30 K cloud peaks at 100 μm, and a 100 K cloud will peak at 30 μm. Hot stars (30000 K) peak at 100 nm in the UV. The Sun (6000 K) peaks at 500 nm in the visible (green).

2. A hotter blackbody has a higher surface brightness intensity at *all* frequencies. This can be seen in Figure 1.

   It is important to remember that more intensity at all frequencies does not necessarily mean more energy! Think about burners on a stove. A small hot burner will have very intense radiation. A large cooler burner will have less intense radiation. But the larger one may boil water faster because although its intensity (surface brightness) is lower, it emits more total energy. What matters is the product of the surface brightness and the emitting area.

   Let's quantify this. To find the intensity (not the specific intensity), we integrate over all frequencies or wavelengths:
$$B(T) = \int_0^\infty B_\nu(T) d\nu\,. \tag{6}$$
After some math, this integral results in the expression
$$B(T) = \frac{\sigma T^4}{\pi}\,, \tag{7}$$
where σ is of course the Stephan-Boltzmann constant. In the case of an isotropic radiation field, which we can frequently assume, it can be shown that $F_\nu = \pi B_\nu$, so therefore $F = \sigma T^4$. This is of course the *Stephan-Boltzmann Law.* We are often interested in the total luminosity of an object (in erg s^{-1} or W):
$$L = \int_S F dA\,, \tag{8}$$
the flux integrated over the emitting surface. For spherical objects, this leads to $L = 4\pi r^2 \sigma T^4$, where r is the object's radius. Thus, the total energy output is related to the surface area and the temperature.

Applications in Radio Astronomy

If we are on the right-hand (long-wavelength) side of the peak, we can Taylor expand the exponential: $e^{hc/\lambda kT} - 1 \simeq 1 + hc/\lambda kT - 1 = hc/\lambda kT$. In frequency units, we find $e^{h\nu/kT} - 1 \simeq 1 + h\nu/kT - 1 = h\nu/kT$. We can therefore write

$$B_\lambda \simeq \frac{2ckT}{\lambda^4} \tag{9}$$

or

$$B_\nu \simeq \frac{2\nu^2 kT}{c^2} \tag{10}$$

This is known as the *Rayleigh-Jeans limit* or *Rayleigh-Jeans approximation*. We almost always assume this limit in radio astronomy. For example,

$$S_\nu = \int_{beam} I_\nu d\Omega \,, \tag{11}$$

where S_ν is the flux density [radio astronomy doesn't use F_ν for some reason], and the integration is over the telescope beam (not necessarily over the entire source!). If we approximate the spectral shape of the source as that of a blackbody, we can refer to the temperature as the *brightness temperature*, T_B, and then

$$S_\nu = \int_{beam} B_\nu d\Omega = \int_{beam} \frac{2\nu^2}{c^2} kT_B d\Omega \,. \tag{12}$$

What is the brightness temperature? It is the value that is needed to give the measured flux S_ν. Wikipedia's definition: "Brightness temperature is the temperature a black body in thermal equilibrium with its surroundings would have to be to duplicate the observed intensity of a grey body object at a frequency ν." This is of course not necessarily the kinetic temperature. *If* the source has a constant surface brightness over the telescope beam,

$$S_\nu = \frac{2\nu^2}{c^2} kT_B \Omega \,. \tag{13}$$

Using Blackbodies

We can usually assume that stars emit similarly to blackbodies, in which case we know their approximate spectral shape for a given temperature. Therefore, observations of stars using astronomical filters can give you information about the temperatures of those stars. Since the temperature and mass are related, we can get a proxy for mass.

The flux (or magnitude) that we measure depends on the filter used. In the optical we may use the U, B, and V filters. We measure the convolution of the filter transmittance and the source spectrum.

Imagine two filters placed on a blackbody curve. The flux ratio of these filters will give you some information about how the spectrum is decreasing. For example, if the flux ratio is large (the longer-wavelength filter is reading much less), the decrease is steep and we must be on the long wavelength side of a high temperature peak. If the flux ratio is small, we must be on the short wavelength side of a low temperature peak. From our discussion of magnitudes, we know that flux ratios are called colors. Colors therefore tell you about the spectral shape, and the temperature of the object.

That colors are useful relies on the fact that stellar spectra are similar to that of blackbodies. This is obvious from Figure 2 below (Figure 3.11 in Carroll & Ostlie), where the U-V and B-V colors of stars are compared to those of blackbodies.

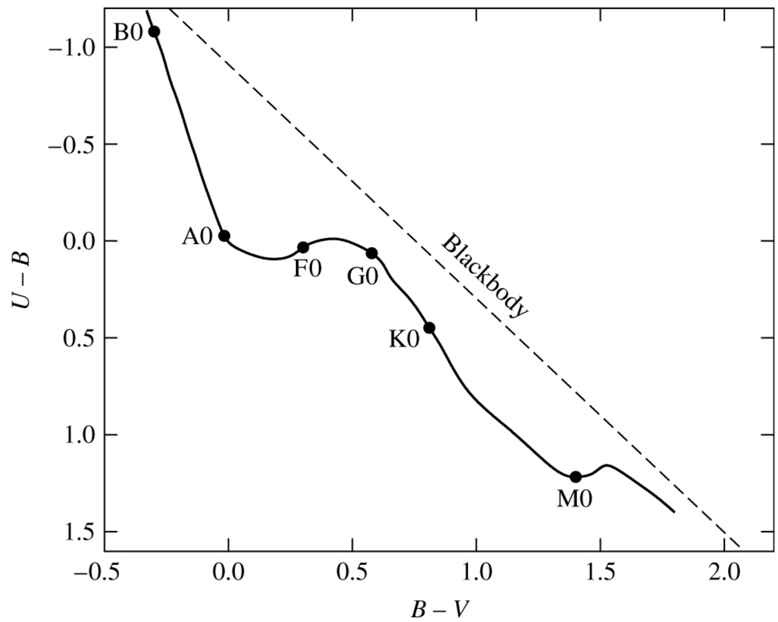

Figure 2: $B - V$ and $U - B$ colors for star of various spectral types. B0 is the largest and M0 are the smallest mass stars in the diagram.

Astronomers use colors as a proxy for temperatures, for example on the color-magnitude diagram, CMD. The CMD looks almost exactly like the H-R diagram because there is such a clean mapping between colors and temperatures. Why use the CMD? The quantities are entirely observable. In the H-R diagram, we often do not know the luminosity and temperature, but we can easily measure magnitudes for a bunch of stars.

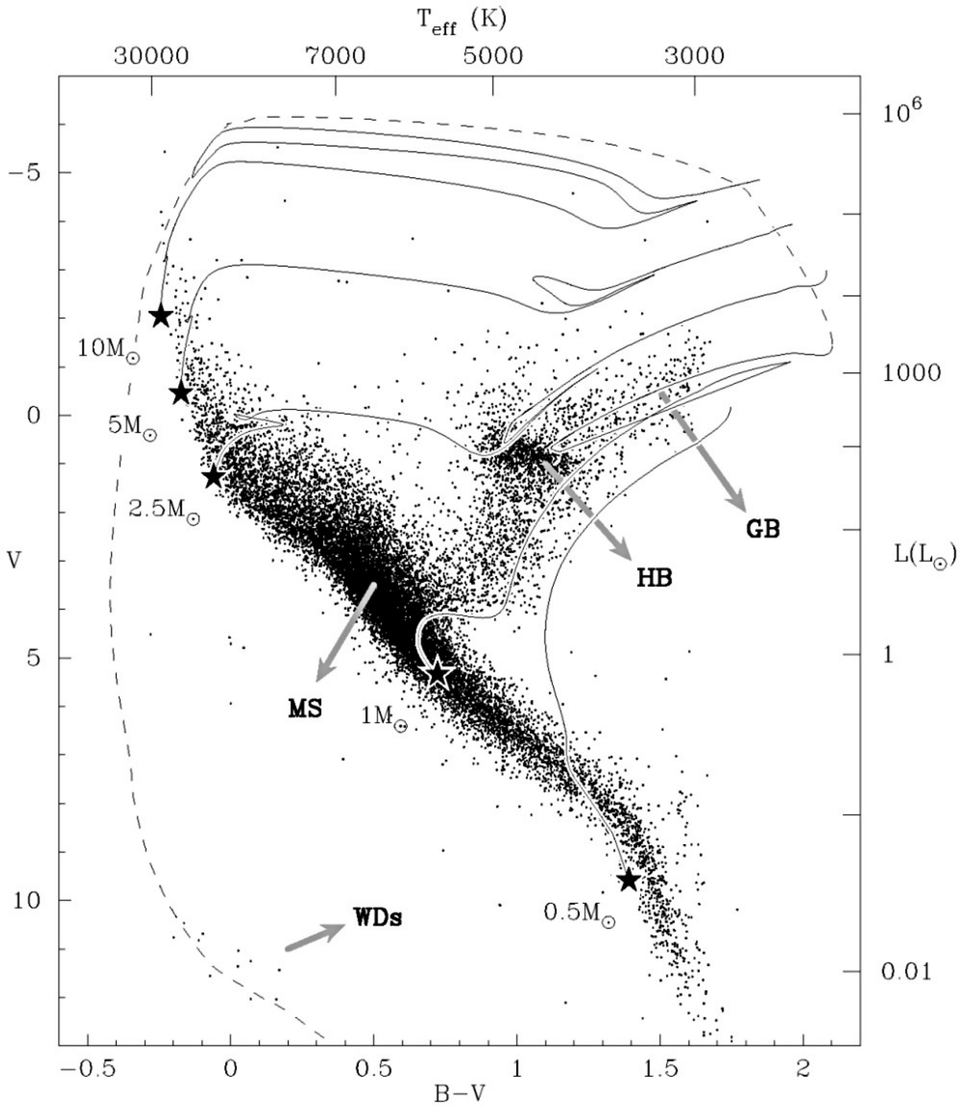

Figure 3: A Color-Magnitude Diagram (CMD) (as in the earlier lecture). Each dot corresponds to one star. Shown are the main sequence (MS), location of white dwarfs (WD), the Horizontal Branch (HB), and the Giant Branch (GB). With time, stars evolve off the main sequence, go up into the giant branch, back down into the horizontal branch, and eventually become white dwarfs. The evolutionary tracks for stars of various masses are also shown.