Stellar Pulsations
C+O Chapter 14

We have discussed stellar pulsations in a few contexts so far. These pulsations give rise to luminosity, and therefore magnitude, fluctuations.

Pulsating stars are important for astronomy. Stellar pulsations reveal physics at work in stellar interiors, and stellar pulsations can even be used to determine distances to stars (although we won't cover that here).

Astronomers have classified a large number of pulsating stars. Although these stars can have different masses and are at different points in their evolutions, we can describe their physics in similar ways.

There are probably several million pulsating stars in the Milky Way, and the total stellar population of the Milky Way is several hundred billion stars. Many pulsating stars live in the "instability strip" in the H-R diagram.

Types of Variable Stars

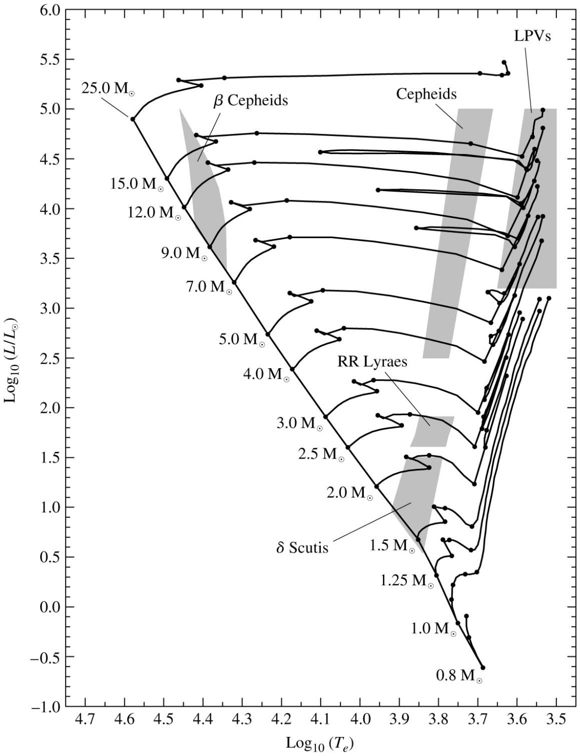

Figure 1: Pulsating star locations on the H-R diagram.

Mira or Long Period Variables

David Fabricius gets credit as the first westerner to note a variable star, that of Mira in Cetus, in 1595. The star he was observing is named “Mira,” and this class of stars is known as “Mira variables,” or “long period variables.” They are bright stars that have periods between 100 and 700 days.

Mira variables are AGB stars undergoing shell H and He fusion. They are variable because such shell fusion is unstable. Although the H-R diagram from the book lists a higher mass, Mira variables should be less than 2 Solar masses.

Stellar Populations

We have talked about metallicity, but astronomers have a shorthand for discussion metallicity that comes up here: that of “populations.”

Population I, or metal-rich, stars are young stars with the highest metallicity out of all three populations. The Sun is population I.

Population II, or metal-poor, stars are those with relatively little of the elements heavier than helium. These objects were formed during an earlier time of the universe.

Population III stars are a hypothetical population of extremely massive, luminous and hot stars with virtually no metals, except possibly for intermixing ejecta from other nearby, early population III supernovae.

Cepheids

Nearly 200 years went by before the next variable star was identified. I honestly don’t understand why it took so long! On September 10, 1784, Edward Pigott detected the variability of Eta Aquilae, the first known representative of the class of classical Cepheid variables. The eponymous star for classical Cepheids, Delta Cephei, was discovered to be variable by John Goodricke a few months later. δ Cephei, varies regularly with a period of 5 days 8 hours and 48 minutes. So-called “Cepheids” are incredibly important for astronomy.

Cepheids vary over periods of $\sim 1 - 100$ days and can be quite luminous. There are tens of thousands known.

Henrietta Swan Leavitt discovered thousands of Cepheids while working as a “computer” at Harvard. Computers were women who did work that men didn’t want to. Her task was to compare photographs to identify variable stars. She then noticed that the most luminous Cepheids have the longest periods, leading to the “period-luminosity relationship,” one of

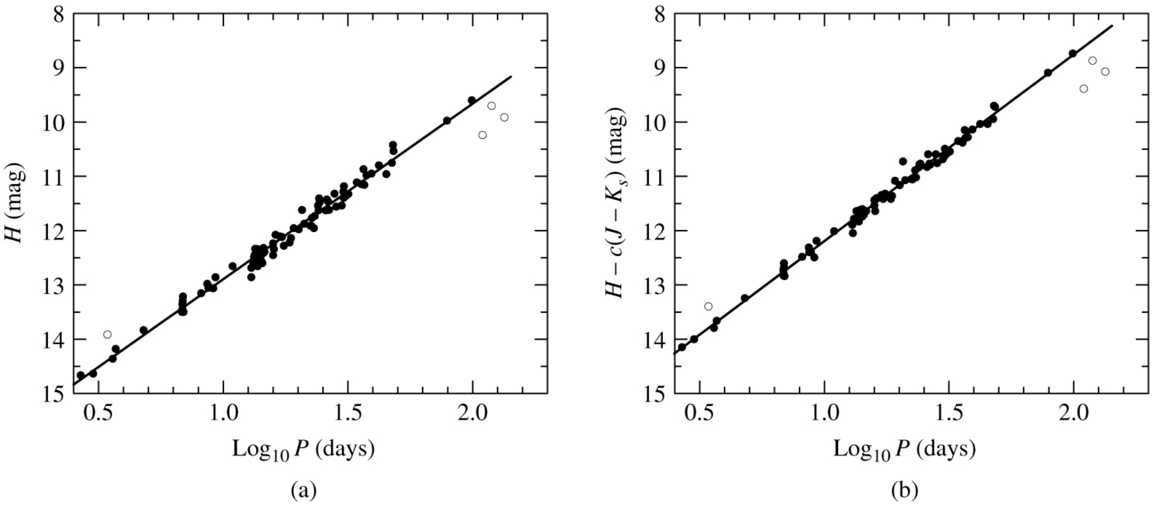

Figure 2: The Period-luminosity relation of classical Cepheids.

the most useful relationships in astronomy.

The period-luminosity relationship can be strengthened by observing in the infrared ("extinction" from dust is less of an issue) or by using only certain Cepheids (it turns out there are different kinds, or even by adding a color term to the fit. The most basic relation is give by your book in V-band as

$$M_V = -2.81 \log_{10} P_d - 1.54 \,, \tag{1}$$

where the period is given in days.

Cepheid variables are divided into two subclasses which exhibit markedly different masses, ages, and evolutionary histories: classical Cepheids and type II Cepheids.

Population I Cepheids (Type I Cepheids or Delta Cepheid variables) undergo pulsations with very regular periods on the order of days to months. Classical Cepheids are Population I variable stars which are 4-20 times more massive than the Sun, and up to 100,000 times more luminous. These Cepheids are yellow bright giants and supergiants of spectral class F6-K2 and their radii change by millions of kilometers during a pulsation cycle.

Classical Cepheids are used to determine distances to galaxies within the Local Group and beyond, and are a means by which the Hubble constant can be established. Classical Cepheids have also been used to clarify many characteristics of the Milky Way galaxy, such as the Sun's height above the galactic plane and the Galaxy's local spiral structure.

A group of classical Cepheids with small amplitudes and sinusoidal light curves are often separated out as Small Amplitude Cepheids or s-Cepheids, many of them pulsating in the first overtone.

Type II Cepheids Type II Cepheids (also termed Population II Cepheids) are population II variable stars which pulsate with periods typically between 1 and 50 days.Type II Cepheids are typically metal-poor, old ($\sim 10\,\mathrm{Gyr}$), low mass objects (about half the mass of the Sun). Type II Cepheids are divided into several subgroups by period. Stars with periods between 1 and 4 days are of the BL Her subclass, 1020 days belong to the W Virginis subclass, and stars with periods greater than 20 days belong to the RV Tauri subclass.

Anomalous Cepheids are a group of pulsating stars on the instability strip have periods of less than 2 days, similar to RR Lyrae variables but with higher luminosities. Anomalous Cepheid variables have masses higher than type II Cepheids, RR Lyrae variables, and the Sun. It is unclear whether they are young stars on a “turned-back” horizontal branch, blue stragglers formed through mass transfer in binary systems, or a mix of both.

Other Types of Variable Stars

Other types of variable stars are: W Virginis stars, RR Lyrae stars, δ Scuti stars, β Cephei stars, and ZZ Ceti stars (and many more!). These have periods ranging from 100s of seconds to days. ZZ Ceti stars are actually pulsating white dwarfs. Cool!

The Physics of Stellar Pulsations

Just as earthquakes tell us about processes in the Earth’s interior, stellar pulsations can reveal processes in stellar interiors.

How fast do stars pulsate?

We can once again arrive at a rough timescale by seeing how long it would take a pressure wave to traverse the medium (in this case, the star). Many processes are bound by this speed, and it therefore gives us a rough way of estimating timescales. Remember, we used this same line of argument when deriving the Jean’s mass.

The sound speed (pressure wave speed) is

$$c_s = \sqrt{\frac{\gamma P}{\rho}}\,. \tag{2}$$

If we assume hydrostatic equilibrium, and further (unrealistically) assume constant density,

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\frac{G\left(4/3\pi r^3\rho\right)\rho}{r^2} = -4/3\pi G\rho^2 r\,. \tag{3}$$

We can integrate this going from the outside in, from $r = R$ to $r = r$ and from $P = 0$ to

$P = P$:

$$\int_0^P dP = \int_R^r -4/3\pi G\rho^2 r dr \tag{4}$$

$$P(r) = 2/3\pi G\rho^2(R^2 - r^2)\,. \tag{5}$$

(note that this is a general expression. We didn't derive it when discussing stellar structure, because it is kind of wrong....) If we define the pulsation period as

$$\Pi \approx 2\int_0^R \frac{dr}{c_s} \approx 2\int_0^R \frac{dr}{\sqrt{2/3\gamma\pi G\rho(R^2 - r^2)}} \approx \sqrt{\frac{3\pi}{2\gamma G\rho}} \propto 1/\sqrt{\rho}\,. \tag{6}$$

So the pulsation period varies as the inverse of the density. Thus, ZZ Ceti stars, with high densities, pulsate rapidly. Note also how similar this is to the free-fall time!

Radial Modes of Pulsation

Stars pulsate radially, in a manner that is similar to that of sound waves in an organ pipe. There is a fixed knot at the star's center, or the closed end of the organ pipe, and an open end at the star's surface, or the open end of the organ pipe. For the fundamental mode, the pressure wave moves through the star radially, starting in the center. For the first overtone, there is a second node that reverses the flow. For the second overtone, there is a second node that reverses it again.

Classical Cepheids and W Viginis stars pulsate in the fundamental mode.

Eddington's Thermodynamic Heat Engine (the "κ-mechanism")

Sir Arthur Eddington (of the Eddington Limit) proposed that variable stars are actually heat engines. A heat engine converts thermal or chemical energy into mechanical energy, like the internal combustion engine of cars.

If gas expands or contracts, it does PdV work. If over a cycle $\int PdV > 0$, the net work is positive and the oscillations can grow in amplitude. If the net work is negative, the oscillations are damped.

The net work done by each layer of the star during one cycle is the difference between the heat flowing into the gas and the heat leaving the gas. For driving, the heat must enter during the high-temperature part of the cycle and leave during the low-temperature part. The driving layers of a pulsating star must absorb heat around the time of their maximum compression. The maximum pressure will occur after maximum compression.

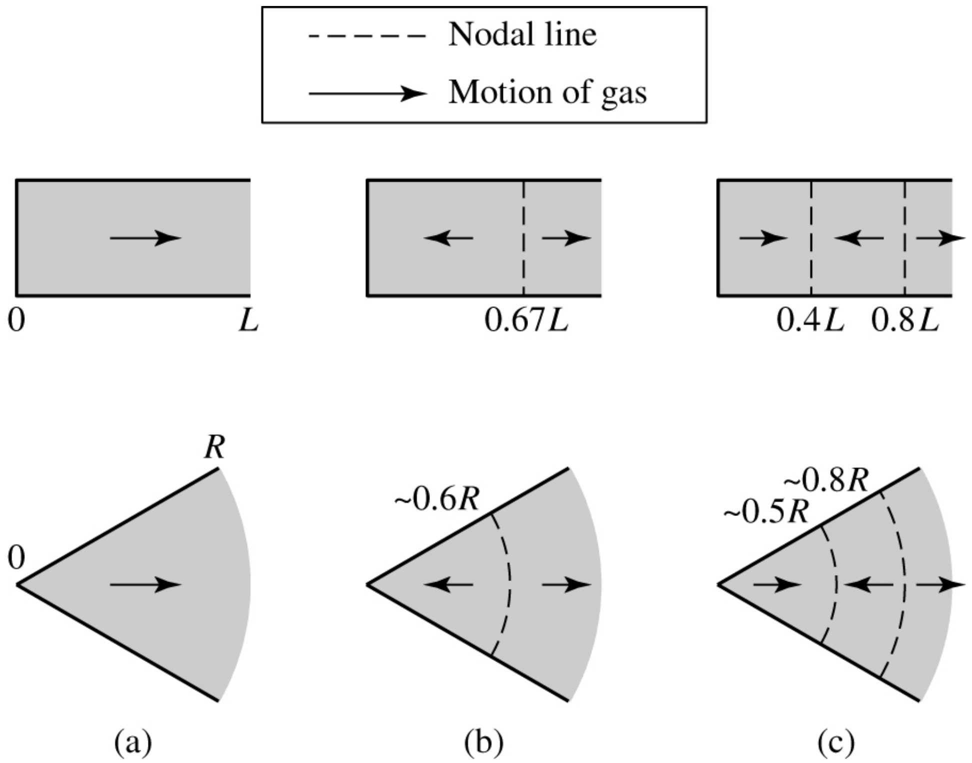

Figure 3: Radial pulsation modes for (L to R) fundamental, first overtone, and second overtones.

Eddington's Valve

Eddington also came up with an explanation for how the heat engine may proceed. It turns out that oscillations are too small in star's cores to explain the wide range of luminosities during pulsation.

If a star becomes more opaque upon compression, energy has more trouble escaping (this is the κ in “κ-mechanism). This energy is trapped within the star, pushing the surface layers outward. The expanding layer becomes more transparent, the heat escapes, the layer falls back down, and the cycle begins again.

This, however, is counter-intuitive. For most stellar material, $\kappa \propto \rho T^{-3.5}$, so if we assume ideal gas, $\kappa \propto \rho^{-2.5}$. We need the opacity to increase as the density increases for the heat engine to be operational. This requires more fine-tuning, which is why most stars don't pulsate.

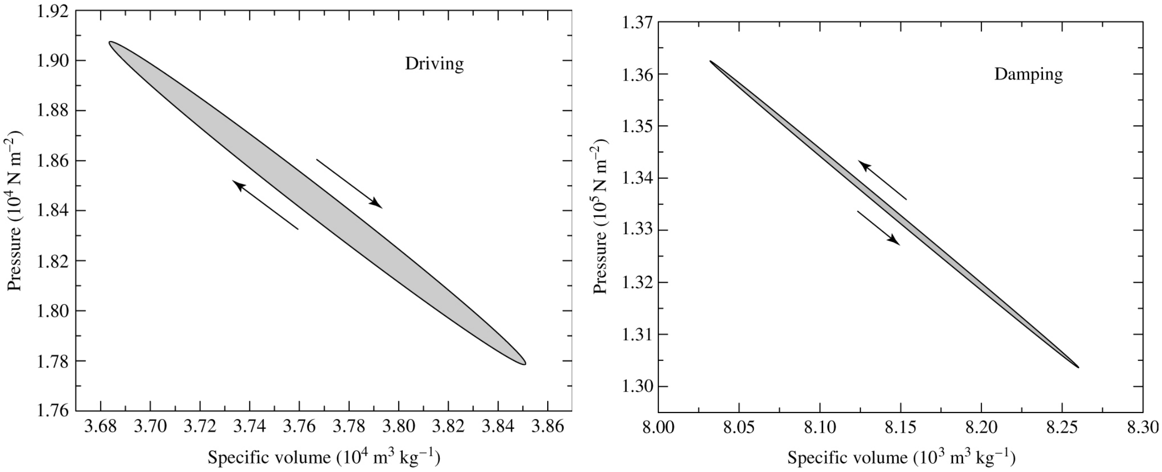

Figure 4: Eddington's Heat Engine. (left) This cycle shows a net driving engine moving in the clockwise direction. (right) This is a net damping engine moving in the counterclockwise direction.

κ and γ Mechanisms

The fine-tuning is actually just partial ionization, which allows stars to absorb energy to flow into a compressed region (the γ mechanism) and for the compressed gas to absorb this heat (the "κ" mechanism). The locations of these partial ionization zones differ depending on the size of the star.

Here, the Greek letter kappa is used to indicate the radiative opacity at any particular depth of the stellar atmosphere. In a normal star, an increase in compression of the atmosphere causes an increase in temperature and density; this produces a decrease in the opacity of the atmosphere, allowing energy to escape more rapidly. The result is an equilibrium condition where temperature and pressure are maintained in a balance.

However, in cases where the opacity increases with temperature, the atmosphere becomes unstable against pulsations. If a layer of a stellar atmosphere moves inward, it becomes denser and more opaque, causing heat flow to be checked. In return, this heat increase causes a build-up of pressure that pushes the layer back out again. The result is a cyclic process as the layer repeatedly moves inward and then is forced back out again.