ASTR469 Lecture 2: Quantifying Light (Ch. 5)

1 Light from astronomical objects

Most astronomical objects emit light that changes as you tune to different frequencies in the electromagnetic spectrum. Today we're going to discuss how astronomers quantify the light that is received on Earth within a particular frequency range $\nu + d\nu$.

We can separate quantities into *intrinsic* properties of the astrophysical sources and *measured* properties that we observe. The difference is that measured properties depend on your distance from the source and the effects of intervening material, whereas intrinsic properties do not. Let's start with intrinsic properties.

2 Intrinsic source properties: Luminosity

Astronomical sources emit light due to various mechanisms (blackbody, synchrotron, free-free emission, and other mechanisms we will later discuss). Those processes produce EM waves, and those waves carry energy away from the source. A single photon has an energy that depends on its frequency: $E = h\nu$.

Luminosity is a fundamental quantity that tells you how much energy per second is leaving the surface an object.

Consider a 100 W light bulb; it emits 100 J s^{-1} of energy. This is its *luminosity*, also sometimes referred to as *power*. We typically use units of erg s^{-1} or W (J s^{-1}). I will mostly stick with the latter because it represents S.I. units. The Solar luminosity is 3.828×10^{26} W, so the Sun's intrinsic energy output is roughly equivalent to that of about 4×10^{24} light bulbs.

OK, but what if we wanted to know how much luminosity was coming out at a given frequency? In this case we use the "spectral luminosity," L_ν, which tells us the luminosity that would be observed if you could only see at a particular frequency. The units of L_ν are W Hz^{-1}. This is what astronomers frequently use, because our observations are usually at a particular frequency.

The "bolometric luminosity" L, which we were previously discussing for the light bulb, is simply the integral of the spectral luminosity over all frequencies:

$$L = \int_0^\infty L_\nu d\nu \ . \tag{1}$$

In astrophysics we rarely use the bolemetric luminosity because our telescopes measure L_ν. If we measure L_ν at a number of frequency bands, we can fit a spectrum that has a known form and physical origin (like a blackbody spectrum, for instance, which we will discuss later). Then, we can integrate that to get the bolometric luminosity.

Side note and sneak-preview... you have probably encountered luminosity before with the

Stephan-Boltzmann law:

$$L = A\sigma T^4 \,, \tag{2}$$

where A is the emitting area, σ is the Stephan-Boltzmann constant, and T is the surface temperature. This equation gives the total luminosity emitted from a blackbody-emitting object. For spherical objects, of course $A = 4\pi R^2$ because the light is going in all directions at the same time. This formula is applicable for stars.

3 Measured Source Properties: Intensity and Flux

3.1 Intensity

Intensity is what telescopes measure. The *spectral or "specific" intensity* of radiation, I_ν, is the most basic observable quantity. Here, *specific* refers to the fact that it is measured at one particular wavelength (thus the ν subscript), similar to *spectral* luminosity. The specific intensity has units of energy, per unit time, per unit area, per unit frequency, per unit *steradian*, or J s^{-1} cm^{-2} Hz^{-1} sr^{-1} (or in wavelength form, J s^{-1} cm^{-2} m^{-1} sr^{-1} or, instead of m, fill in whatever wavelength units you want to use: nm, Angstrom, etc.).

The specific intensity goes by many other names, which can make things even more confusing; sometimes people call it radiance, irradiance, brightness, or surface brightness. Annoyingly, sometimes people also say "brightness" and mean "flux". It's kind of a mess.

Two important things about the specific intensity:
(1) It is independent of distance if the light travels through free space. Thus, the camera exposure time and aperture setting for an exposure of the Sun would be the same, regardless of whether the photograph was taken close to the Sun (from near Venus, for example) or far away from the Sun (from near Mars, for example), so long *as the Sun is resolved* in the photograph. This seems terribly wrong at first, but can easily be proven.
(2) Somewhat related to the previous point, it is the same at the source and at the detector. Thus you can think of brightness in terms of energy flowing out of the source or as energy flowing into the detector. Those two quantities will be the same.

We can conceptualize intensity as the energy dE passing through an infinitesimally small area dA by:

$$dE = I_\nu \, dA \, \cos\theta \; d\Omega \, d\nu \, dt \,, \tag{3}$$

where θ is measured normal to the surface dA and $d\Omega$ is the solid angle. (For our measurements, we can safely disregard θ since we are almost always observing normal to the detector.) We can rearrange to find:

$$I_\nu = \frac{dE}{dA \, \cos\theta \, d\Omega \, d\nu \, dt} \,. \tag{4}$$

Notice that we wrote the spectral intensity in frequency units. I_ν has a dependence on $d\nu$, and $d\nu \neq d\lambda$. Instead, $c = \lambda\nu$ so

$$d\nu = -(c/\lambda^2) d\lambda \tag{5}$$

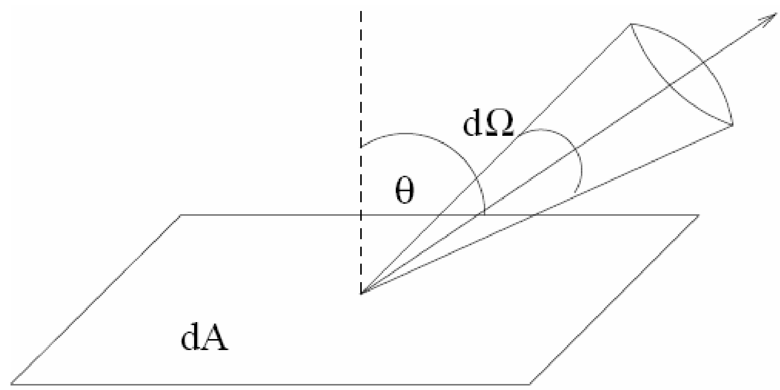

Figure 1: The geometry for spectral intensity.

so combining with the above equations

$$\nu I_\nu = \lambda I_\lambda \tag{6}$$

To get the *intensity* or *integrated intensity* we would of course integrate over frequency or wavelength:

$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda \tag{7}$$

As for bolometric luminosity, this is rarely done.

3.2 Flux

While intensity is perfect for extended sources, we are frequently more interested in the quantity of *flux* integrated over solid angle:

$$F_\nu = \int I_\nu \cos\theta d\Omega \tag{8}$$

or

$$F_\nu = \int_0^{2\pi} \int_0^\pi I_\nu \cos\theta \sin\theta d\theta d\phi\,. \tag{9}$$

The units of flux are therefore $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$ or $\mathrm{W\,m^{-2}\,Hz^{-1}}$. The integration is carried out over the solid angle of the observations (not the source).

Why do we care about flux when we have a perfectly good unit of specific intensity? Remember that specific intensity is a surface brightness. The source may not be *resolved*, that is, it may effectively be a single point of light. This is true for stars, for example, which are *unresolved* with nearly all telescopes. So, for small, unresolved *point sources*, the flux is a much better measure. For large, resolved sources the spectral intensity is generally more useful.

We can relate the flux and luminosity with

$$L_\nu = 4\pi d^2 F_\nu\,, \tag{10}$$

where d is the distance to the source.

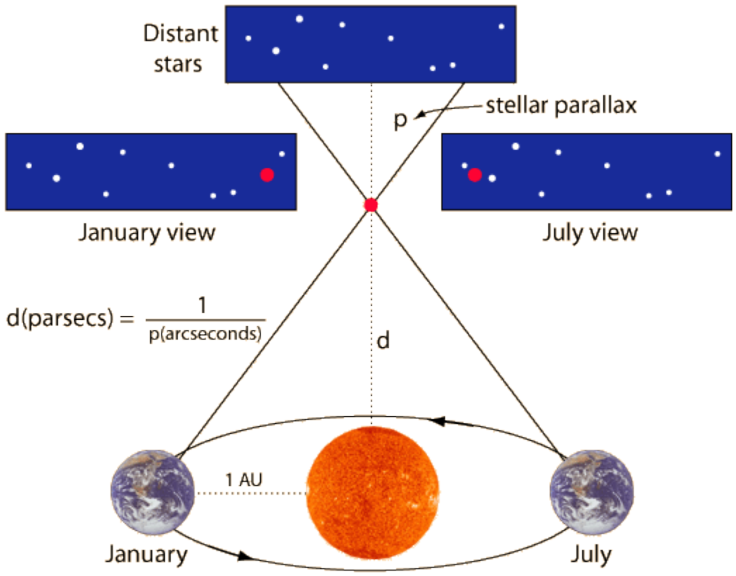

Figure 2: The definition of parallax.

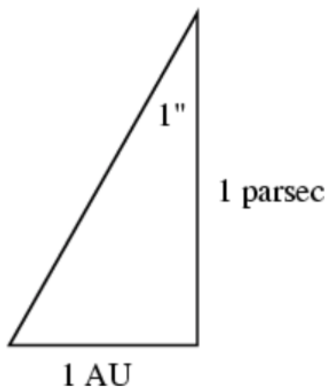

Figure 3: The definition of a parsec. Note that this implies $\tan(1") = \frac{1\mathrm{AU}}{1\,\mathrm{pc}}$, which is a convenient thing to remember if you forget the conversion from parsecs to meters.

4 Astronomical Distances and Source Sizes

Space is big and astronomers use special (non-S.I.) units for distances. The most useful is the *parallax*, which is the distance at which the apparent position of a star changes by one arcsecond (1/3600 of a degree) as the Earth goes around the Sun.

The equation is therefore $d = 1/p$ for d measured in parsecs and p measured in arcseconds. We define one "Astronomical Unit", 1 AU, as the distance between Earth and the Sun. The definition is fixed at $1\,\mathrm{AU} = 1.496 \times 10^{11}$ m. Therefore,

$$1\,\mathrm{pc} = 3.08 \times 10^{16}\,\mathrm{m} \quad (11)$$

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Assess yourself/study guide after lecture & reading (without peeking at notes)...

1. What measure would you use to determine the energy output of a source, and what are its units?

2. What are the units of the energy that hits a detector (flux)? [Note: your detector has a finite size.]

3. You're looking at an object with constant intensity as a function of angular position. It appears round, with an angular radius of 4". You measure its spectral intensity as $10^4\,\mathrm{W\,Hz^{-1}\,m^1\,sr^{-1}}$. What is the total flux detected from the whole object?

4. You observe the flux of a point-source object in the distance to be $5\ \mathrm{W\,m^{-2}\,Hz^{-1}}$. If it is $5\,\mathrm{km}$ away, what is the luminosity of the source?

5. Let's say you have a detector operating at $1\,\mathrm{THz}$ frequency. The detector has a collecting area of $2\,\mathrm{m^2}$ and is pointed directly at the target. How many photons per second at that frequency are you detecting from an object of $F_{1THz} = 10^{-20}\,\mathrm{W\,m^{-2}\,Hz}$?