

ISM HW #1

1) $M = 7 \times 10^9 M_\odot$

$R_{\text{disk}} = 15 \text{ kpc}$

$h = 200 \text{ pc}$

$m_{\text{fp}} = \frac{1}{n\sigma}$

$$n = \frac{N}{V} = \frac{7 \times 10^9 M_\odot / m_H}{\pi \cdot (15 \text{ kpc})^2 \cdot 200 \text{ pc}} = \frac{8.37 \times 10^{66}}{4.15 \times 10^{66} \text{ cm}^3}$$

$= 2.00 \text{ cm}^{-3}$

$\sigma \approx \pi \cdot (5.29 \times 10^{-9} \text{ m})^2$
 $= 8.79 \times 10^{-17} \text{ cm}^2$

(Approx. real values
 from QM; this is Bohr rad.)

$m_{\text{fp}} = \frac{1}{8.79 \times 10^{-17} \text{ cm}^2 \cdot 200 \text{ cm}^{-3}} = 5.6 \times 10^{15} \text{ cm} = 0.018 \text{ pc}$

b) $M_{\text{clouds}} = 0.3 \times 7 \times 10^9 M_\odot = 2.1 \times 10^9 M_\odot$

$M_{\text{cloud}} = n \cdot m_{\text{H}_2} \cdot V = 100 \text{ cm}^{-3} \cdot 3.35 \times 10^{-24} \text{ g} \cdot \frac{4}{3} \pi (15 \text{ kpc})^3$
 $= 1.39 \times 10^{38} \text{ g} = 7.00 \times 10^4 M_\odot$

$N = M_{\text{clouds}} / M_{\text{cloud}} = 3 \times 10^4$

Assume same density. New mass per particle increases

$\mu = \frac{2 \cdot m_{\text{H}} + 2 \cdot m_{\text{He}} \cdot 0.1}{m_{\text{H}}} = 1.4$

So mass of cloud increases by 40%.

N decreases by 40%.

$$2) \rho_{\text{grain}} = 2 \text{ g cm}^{-3}$$

$$a_{\text{grain}} = 0.1 \mu\text{m}$$

$$m_{\text{grain}} = \rho_{\text{grain}} \cdot V_{\text{grain}} = 2 \text{ g cm}^{-3} \cdot \frac{4}{3} \pi (0.1 \mu\text{m})^3$$
$$= 8.4 \times 10^{-15} \text{ g}$$

Spacecraft sweeps through volume

$$V = A \cdot v \cdot t$$

$$= A \cdot 26 \text{ km/s} \cdot 1 \text{ hr}$$

$$= A \cdot 9.4 \times 10^9 \text{ cm}$$

For one particle per hour,

$$n_{\text{dust}}^{-1} = V = 9.4 \times 10^9 \text{ cm} \cdot A$$

$$\Rightarrow A = \frac{n_{\text{dust}}^{-1}}{9.4 \times 10^9 \text{ cm}}$$

Just need n_{dust}

$$\rho_{\text{dust}} = 0.005 \rho_{\text{gas}} = 0.005 (\rho_{\text{H}} \rho_{\text{He}})$$

$$\mu = \frac{m_{\text{H}} + m_{\text{He}} \cdot 0.1}{m_{\text{H}}} = 1.4$$

$$\rho_{\text{dust}} = 0.005 (1.4 \rho_{\text{H}}) = 0.005 \cdot 1.4 \cdot 0.72 \text{ cm}^{-3} \cdot \text{MH}$$

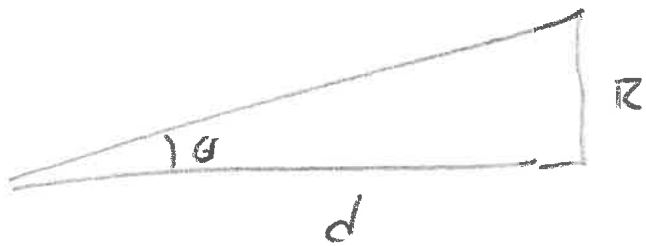
$$\Rightarrow n_{\text{dust}} = \rho_{\text{dust}} / m_{\text{grain}} = 3.1 \times 10^{-13} \text{ cm}^{-3}$$

$$\Rightarrow A = 347 \text{ cm}^2$$

$$3) P = nkT = 30 \text{ cm}^{-3} k \cdot 10^2 \text{ K} = 4.141 \text{ erg cm}^{-3}$$

All in equilibrium, so $n = 0.3 \text{ cm}^{-3}$ $n = 3 \times 10^3 \text{ cm}^{-3}$

$$4) \Omega = \pi \theta^2$$



$$\theta = \frac{R}{d}$$

[from wikipedia]

$$R = 1.22 R_{\odot} = 8.49 \times 10^{11} \text{ cm}$$

$$d = 1.33 \text{ pc} = 4.10 \times 10^{18} \text{ cm}$$

$$\theta = 2.07 \times 10^{-7} \text{ rad}$$

$$\Omega = \pi \theta^2 = 1.35 \times 10^{-13} \text{ sr}$$

$$\Omega = 2\pi(1 - \cos\theta) = 1.35 \times 10^{-13} \text{ sr}$$

Differ by $\frac{\Delta\Omega}{\Omega} = 3 \times 10^{-15}$