ASTR705 ISM
HW #5
Due Friday, 2/24 in class

1) You can use ratios of the same transition of different isotopologues to derive the optical depth and column density of molecular gas. Assume we are observing a molecular cloud with plane-parallel geometry and constant excitation temperature such that

$$I_\nu - I_{\nu,BG} = (S_\nu - I_{\nu,BG})(1 - e^{-\tau_\nu}) , \qquad (1)$$

where $I_{\nu,BG}$ is the intensity of background radiation and S_ν is the source function. Do not make the Rayleigh-Jeans approximation so that terms look like

$$\frac{1}{e^{h\nu/kT} - 1} \qquad (2)$$

and make the substitution

$$T^* = \left[\frac{1}{e^{h\nu/kT_{\rm ex}} - 1} - \frac{1}{e^{h\nu/kT_{BG}} - 1} \right] \frac{h\nu}{k} , \qquad (3)$$

where T^* is what is measured by your telescope, to
a) (3 pt) Derive an expression for the excitation temperature in terms of τ_ν, T^*, and T_{BG}
b) (3 pt) Derive an expression for the optical depth of ^{13}CO using the ratio $T^*_{12\rm CO}/T^*_{13\rm CO}$
c) (3 pt) Use this expression to derive the column density of ^{13}CO assuming $T_{\rm ex} = 10$ K for both species. Use the $J = 1 \to 0$ transition.

2) By observing multiple rotational transitions of a single molecule, you can derive the excitation temperature. We derived this in class, so please refer to your notes.

a) (3 pt) What is the excitation temperature of CO, assuming you have measured the following column densities:

J	N_J
$1 \to 0$	7×10^{15}
$4 \to 3$	5×10^{14}
$7 \to 6$	2×10^{12}
$10 \to 9$	1×10^{9}

While it may seem strange to use column densities here, their use follows naturally from the equation, and in the optically thin limit $T_B \propto \tau \propto N$.

b) (2 pt) Can the data be characterized by a single value of $T_{\rm ex}$? What value do you find?

3) a) (2 pt) Calculate the collisional rate of CO with molecular hydrogen by assuming that the CO cross section is $\sigma = \pi r^2$, where $r = 1.128$ Angstroms. Assume a typical molecular cloud temperature of 10 K. Leave the density term in your answer.

b) (2 pt) What is the critical density of the CO $J = 1 \rightarrow 0$ transition? What is the critical density of the CO $J = 7 \rightarrow 6$ transition? The Einstein As are: $A_{10} = 7.26 \times 10^{-8}\,\mathrm{s}^{-1}$ and $A_{76} = 2.83 \times 10^{-5}\,\mathrm{s}^{-1}$.
c) (3 pt) Very roughly, plot the excitation temperature as a function of density, for both transitions.
d) (3 pt) Plot the fraction of molecules in the $J = 1$ and the $J = 7$ states assuming LTE (Wolfram Alpha works for this). At what temperatures are the populations in each state at a maximum? How does this temperature relate to the rotational energy of the state above the ground (in temperature units $T = E/k$) for the two transitions? Note: the degeneracy of state J is $g_J = 2J + 1$.