ISM HW #8

1) $\Delta t(1420 \text{ MHz}, 1610 \text{ MHz}) = 0.0913 \text{ s}$

Draine eqn. 11.11

$$\frac{dt_{arr}}{d(\nu_9^{-2})} = 4.146 \times 10^{-3} \frac{DM}{cm^{-3}\,pc}$$

$$\Rightarrow DM = \frac{\Delta t}{4.146 \times 10^{-3} \left(1/1.42^2 - 1/1.61^2 \right)}$$

$$= 100 \text{ cm}^{-3} \text{ pc}$$

b) $DM = \int_0^L n_e \, dl \hat{=} \langle n_e \rangle L$

$L = 6000 \text{ pc} \Rightarrow \langle n_e \rangle = 0.03 \text{ cm}^{-3}$

2) $\nu_1 = 1610$ MHz $= 18.63$ cm

$\nu_2 = 1660$ MHz $= 18.07$ cm

$\Delta\psi = 57.5° = 1$ rad

$$RM = \frac{\Delta\psi}{\Delta\lambda^2} = \frac{1 \text{ rad}}{(18.63\text{ cm})^2 - (18.07\text{ cm})^2}$$

$$= 4.87 \times 10^{-2} \text{ rad cm}^{-2}$$

There is a π ambiguity, so ψ could be $\psi = 57.5° - 180° = -122.5°$. This, however leads to $|RM| = 0.104$ rad cm^{-2}

b) Next largest at $\Delta\psi = 57.5° + 180° = 4.14$ rad

$\Rightarrow RM = 0.202$

c) Draine Equation 11.24

$$\frac{\langle B_\parallel \rangle}{\mu G} = \frac{4.87 \times 10^{-2}}{8.12 \times 10^{-5} \cdot 200} = 3.00$$

3) Orion peak $EM = 5\times10^6\ cm^{-6}\ pc$

$\Delta V_{H90} = 25\ km/s$

Equal parts thermal + turbulent broadening, so

$$\Delta V_{H90} = (\Delta V_{therm}^2 + \Delta V_{turb}^2)^{1/2}$$

$$\Delta V_{therm} = \Delta V_{turb}$$

$$\Rightarrow \Delta V_{therm} = \frac{25\ km/s}{\sqrt{2}} = 17.7\ km/s$$

c) $\tau_{ff} = \int \kappa_{ff}\, ds$

$$\kappa_{ff} = 1.091\times10^{-25}\ \underbrace{Z_i^{1.882}}_{1}\ T_4^{-1.323}\ \nu_9^{-2.118}\ n_e n_i \quad cm^{-1}$$

We need T_4:

$$\Delta V_{therm} = \sqrt{\frac{8kT\ln 2}{mc^2}}\ \nu_0 = 21.41\left(\frac{T_4}{M/m_H}\right)^{1/2}$$

$$\Rightarrow T = \frac{\Delta V_{th}^2}{21.47^2} = 6780\ K$$

$\nu_9 = 30$

$$\Rightarrow \tau_{ff} = 1.091\times10^{-25}\cdot 0.678^{-1.323}\cdot 30^{-2.118}\cdot 5\times10^6\ \underbrace{3.08\times10^{18}}_{pc\to cm}$$

$\tau_{ff} = 1.25\times10^{-3}$

b) @ 21cm = 1.4 GHz, $\left(\frac{1.4}{30}\right)^{-2.118} = 659$

$\tau_{ff,21} = \tau_{ff,30} \cdot 659 = 0.800$

c) 2.5' diameter, 500pc distant

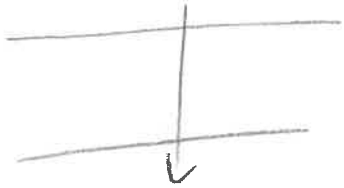
$\Rightarrow 500 \cdot 2.5/60/180 \cdot \pi = 0.36\,pc$ diameter

$EM = \int n_e^2\, ds \approx \langle n_e^2 \rangle \cdot s$

$\Rightarrow \langle n_e \rangle \approx \left(\frac{5 \times 10^6}{0.36}\right)^{1/2} = 3.7 \times 10^3\ cm^{-3}$

Assumes constant density, spherical region

4)



Draine 10.27

$$A_{n\alpha} \approx 6.13 \times 10^9 (n+0.7)^{-5}\ s^{-1}$$

Draine 3.45

$$N_n = n_e N(H^+) \frac{n^2 h^3}{(2\pi m_e kT)^{3/2}} b_n e^{I_n/kT}$$

Optical depth

$$\tau = \sigma_{\ell u} \left(1 - \frac{n_u g_\ell}{n_\ell g_u}\right) N_n$$

$$1 - \frac{n_u g_\ell}{n_\ell g_u} = \beta_n \left(1 - e^{-h\nu/kT}\right) \approx \beta_n\, h\nu/kT$$

Draine 10.26 $\quad \nu \approx 6.577 \times 10^{15} (n+0.5)^{-3}\ s^{-1}$

Draine 6.18 $\quad \sigma_{\ell u} = \frac{\lambda^2}{8\pi} \frac{g_u}{g_\ell} A_{u\ell} \varphi_\nu$

$g = n^2$ so $\frac{g_u}{g_\ell} = \left(\frac{n+1}{n}\right)^2 \approx 1$

$$\varphi_\nu = \frac{c}{\sqrt{2\pi}} \frac{1}{\sigma_v \nu_{n\alpha}} e^{-(v^2/2\sigma^2)}$$

$$= \frac{c}{\sqrt{2\pi}} \frac{1}{\sigma_v \nu_{n\alpha}}$$ at line center

$$\rightarrow \tau_{n\alpha} = \frac{c^2 (n+0.5)^6 \cdot 6.13\times10^9 (n+0.7)^{-5}}{8\pi \cdot (6.13\times10^{15})^2} \cdot \frac{c}{\sqrt{2\pi}} \frac{1}{\sigma_\nu}$$

$$\beta_n \frac{h\nu}{kT} n_e N(\mathrm{H}^+) \left(\frac{n^2 h^2}{2\pi m_e kT}\right)^{3/2} b_n \underbrace{e^{I_n/kT}}_{\approx I_1/kT}$$

whew! Let's put all constants into term C.

$$\tau_{n\alpha} = C \underbrace{(n+0.5)^6}_{\approx n^6} \underbrace{(n+0.7)^{-5}}_{\approx n^{-5}} n^2 \frac{1}{\sigma_\nu} \beta_n b_n \frac{1}{T^{5/2}} EM$$

$$= C\, n^3 \frac{1}{\sigma_\nu} \beta_n b_n \frac{1}{T^{5/2}} EM$$

Carefully evaluating constants finds

$C = 1.205\times10^{-33}$

$$\rightarrow \tau_{n\alpha} = 1.205\times10^{-33} n^3 \beta_n b_n EM\, \sigma_\nu^{-1} T_4^{-5/2}$$

b) $n = 166$ $\quad EM = 10\ \mathrm{cm^{-6}\,pc}$ $\quad b_n \approx 0.9$

$\beta_n \approx -100$

$$\tau_{166\alpha} = 1.205\times10^{-33} \cdot 166^3 \cdot -100 \cdot 0.9 \cdot 10 \cdot \sigma_5^{-1} T_4^{-5/2} \times 3.08\times10^{18}\ \mathrm{cm/pc}$$

$$= -1.53\times10^{-5}\ \sigma^{-1} T_4^{-5/2}$$

$\nu_{166} \approx 6.577\times10^6 (166.5)^{-3} = 1.425\ \mathrm{GHz}$