ASTR368
Gravitational Lensing
Chapter 28.4

Snell's Law

When light passes from one medium into another, it will be bent

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)\,, \tag{1}$$

where n is the index of refraction. Each material has a different index of refraction. This is the ratio of the velocity of light in a vacuum to that in the medium, $n = c/v$. Therefore different materials will bend light different amounts.

Lenses do two things: they bend and magnify light. Lenses have a focal length f.

Lenses will magnify light. We know that from the "thin lens formula":

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}\,, \tag{2}$$

where d_i is the image distance and d_o is the object distance. The magnification

$$M = -\frac{d_o}{d_i} = \frac{f}{f - d_i}\,, \tag{3}$$

where d_i is positive on the left hand side of the image and d_o is positive on the right hand side. If an object is placed at a distance $d_i > f$ from a lens of focal length f, an image will be formed at d_o. In this case both d_i and d_o are positive, and the magnification is negative. This is how a camera works. If the object is at distance $d_i < f$, d_i is negative and the magnification is positive. This is how a magnifying glass or telescope work.

Gravitational Lenses

From Einstein's theory of general relativity, we know that gravity distorts spacetime. Therefore, gravity itself can act as a lens, an effect Einstein noted in 1936 (he said that this would never be observed because the effect was too weak; stupid Einstein). Fritz Zwicky found that galaxy clusters can lens background galaxies in 1937, and the effect was detected in 1979. Gravitational lenses are different from optical lenses because the "lens" is spherical:
1) maximum bending is found close to the lens
2) there is no single focal length, but rather a focal line.
3) because of the large distances and strong gravity, there is a time-delay for less heavily-lensed background events relative to heavily lensed events (this effect is present for normal lenses, but is difficult to observe).

Unlike normal lenses, all light is affected equally for gravitational lenses.

From GR we know that the coordinate speed of light passing distance r from point mass M in the radial direction is

$$\frac{dr}{dt} = c\left(\frac{1\text{–}2GM}{rc^2}\right) \tag{4}$$

So therefore

$$n = \frac{c}{dr/dt} = \left(\frac{1\text{–}2GM}{rc^2}\right)^{-1} \simeq 1 + \frac{2GM}{rc^2} \tag{5}$$

This approximation is valid because $2GM/rc^2$ is so small. Our book notes that 10^4 pc from a 10^{11} $M_\odot$ galaxy $n = 1 + 9.6 \times 10^{-7}$ – pretty much unity.

From GR, the angle of deviation

$$\phi = \frac{4GM}{r_0 c^2} \, , \tag{6}$$

where ϕ is in radians and r_0 is the distance from the mass (this differs from Newtonian by a factor of 2).

In the figure, θ is where the lensed object will appear. From trigonometry, one can find

$$\theta^2 - \beta\theta - \frac{4GM}{c^2}\left(\frac{d_s - d_L}{d_s d_L}\right) = 0 \tag{7}$$

This is obviously a quadratic equation with two solutions for θ:

$$\theta = 0.5\left(\beta \pm \left[\beta^2 + \frac{16GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)\right]^{0.5}\right) \tag{8}$$

For the special case of perfect alignment, $\beta = 0$ and

$$\theta = \left[\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)\right]^{0.5} = \theta_E \, , \tag{9}$$

where θ_E is the "Einstein radius." For cosmological distances, the Einstein radius is $\theta_E \simeq (\text{few}\, M_\odot^{0.5})''$.

We can therefore reformulate the above equation to be

$$\theta^2 - \beta\theta - \theta_E = 0 \tag{10}$$

Because cluster mass distributions are spread out, we can get a variety of morphologies:
1) two images of the background source, as implied by above equations
2) if we do not assume spherical symmetry, we can get an Einstein cross
3) if alignment is perfect, get Einstein ring. Note that in this case we can directly determine the lensing mass if the distances are known. [how can we get distances?]
4) Similar to 3), we find multiple arcs around galaxy clusters. These are partial Einstein rings caused by clumpiness in the gravitational potential

Magnification
The derivation is a bit beyond the level for this course and not terribly instructive, but gravitational lenses result in a magnification that is always > 1.

Time Delay
Because of the longer pathlengths and the fact that the different rays have traversed different potentials, we get path delays of months to years between the same event.

Gravitational lenses have been used to estimate the masses of clusters, which has led to their large inferred mass to light ratios. By examining the position and magnification of the various lensed images, we can reconstruct the mass distribution.