Stellar
Nuclear Fusion

Nuclear Reactions

We know at this point that stars create energy via nuclear fusion, but we haven't yet investigated the processes. Fusion reactions combine two (or more) reactants into one (or more) products. In the process, mass is converted to energy via $E = mc^2$. Therefore, mass is not conserved.

We wish to initially answer two questions:
1) What determines the rate at which stars create energy? and
2) What determines the elements involved?

But first, a review of terms:
q - the rate of energy release per unit mass. Has units of energy/time/mass, or J/s/kg.
Q_{ijk} - the amount of energy released (with reactants i,j,k). Units of energy, e.g. J.
R_{ijk} - the volume reaction rate in units of length^3 time^{-1}, Combining these, we have

$$q = QR/\rho = \left[\frac{\mathrm{J} \times \mathrm{m}^3\mathrm{s}^{-1}}{\mathrm{kgm}^{-3}} = \mathrm{J/s/kg}\right] \tag{1}$$

I thought it would be useful to first review our elementary particles

Fermions are particles with half-integer spins. These include quarks and leptons. Within leptons we have electrons (charge $-e$) and neutrinos (no charge). We also have composite leptons made up of fundamental leptons. Hadrons are examples of these, such as protons (charge $+e$) and neutrons (no charge).

Finally, there are antiparticles with the same spin but opposite charge, for all fundamental particles.

Conservation Laws

Although mass isn't conserved in nuclear fusion, we do have to conserve charge and lepton number. The total number of matter leptons minus the total number of antimatter leptons must be constant.

Binding Energy

The binding energy is the amount of energy required to separate a particle from a system of particles or to disperse all the particles of the system. It is a fundamental parameter of fusion.

A number of key points about the binding energy curve:

1. He^4 is higher than the elements around it
2. Where there is a sharp rise, fusion of similar nuclei will produce a lot of energy
3. The peak of the curve is iron.
4. There are no stable configurations for $A = 5$ or $A = 8$

Why is this chart important? "The energy released in a nuclear reaction Q_{ijk} is a measure of **the difference between the binding energies** of the reactants and the products." So for fusion, it's not the individual binding energy that's important, but the difference in binding energy between the reactants.

Reaction Rates and the Gamow Peak

How can we determine a nuclear reaction rate? How can we even get two particles close together? Two positively-charged nuclei will want to repel each other. To fuse, particles must overcome this "Coulomb barrier." We can define the distance two particles must pass to overcome the Coulomb forces by equating the kinetic, $E_k = 1/2mv^2$ and Coulomb $E_c = Z_i Z_j \mathrm{e}^2/(4\pi\epsilon_0)$ energies. Doing so, we find

$$d = \frac{1}{4\pi\epsilon_0}\frac{Z_i Z_j e^2}{1/2mv^2}\,. \tag{2}$$

Particles must be within the Coulomb barrier to fuse.

For typical temperatures (and velocities), the barrier distance is 3 orders of magnitude more than the typical range of the strong force! That means that using classical physics, we cannot get two ions close enough to fuse. The solution is "tunnelling." George Gamow calculated the tunnelling probability and found that the cross section is proportional to

$$\sigma \propto e^{-\pi Z_i Z_j e^2/(\epsilon_0 h v)} \tag{3}$$

So the volume rate is proportional to the cross section times the velocity, σv, or $R = \sigma v$ (book uses ς). What is v? A Maxwellian! We know therefore that

$$v \propto \mathrm{e}^{-mv^2/(2kT)} \tag{4}$$

m here is the reduced mass since both particles in a given reaction are in motion.

The cross section is harder. We cannot treat particles like billiard balls, and instead must do the full QM calculations. Doing so, we arrive at $\sigma(E)$, the cross section as a function of energy. These functions are different for each species and are approximately log-normal (Gaussian in log-space). We can approximate it

$$\sigma(E) = \frac{S_0}{E} \exp(-E_G/E)\,, \tag{5}$$

with S_0 a constant and E_G the "Gamow Energy."

To calculate the reaction rate, we would have to integrate the product σv over all velocity values:

$$R = \langle \sigma v \rangle = \int_0^\infty \sigma(E) v f(E) dE\,, \tag{6}$$

and $f(E)$ can be described by the M-B distribution written in terms of energy:

$$f(E)dE = \frac{2}{\pi^{1/2}} (kT)^{-3/2} E^{1/2} \exp(-E/kT) dE \tag{7}$$

It can be shown that the value of the integral, and with it the reaction rate, is proportional to the maximum of the product, which occurs for

$$v = (\pi Z_i Z_j e^2 kT/\epsilon_0 h m_g)^{1/3} \tag{8}$$

and therefore the reaction rate

$$R_{\rm ijk} = \sigma v \propto (kT)^{-2/3} \exp\left[-\frac{3}{2}\left(\frac{\pi Z_i Z_j e^2}{\epsilon h}\right)^{2/3}\left(\frac{m_g}{kT}\right)^{1/3}\right] \tag{9}$$

increases with increasing temperature and decreases with increasing charges of the interacting particles. Fusion of heavier and heavier nuclei would therefore require higher and higher temperatures.

From the earlier relations, this has a very strong dependence on temperature. We can write

$$q = q_0 \rho T^n\,, \tag{10}$$

where n is a positive constant (not the density!).

What determines the rate at which stars create energy? Density (to 1st power for most reactions) and temperature (possibly to a higher power). Both of these are determined by the mass of the star.
What determines the elements involved? The difference in binding energy! Differences are largest for the smallest elements.

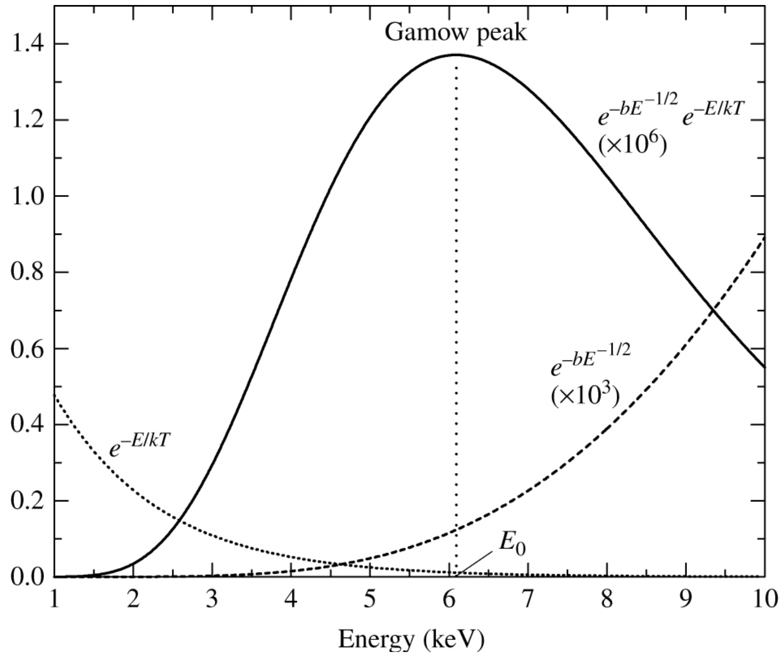

Figure 1: Nuclear reaction probability for proton-proton.

Hydrogen fusion

Until the end of their lives, stars make energy by fusing hydrogen into helium. The process by which they do this, however, is multi-faceted, and depends on stellar temperature.

Proton-Proton Chain

In the normal main sequence portion of a star's life, it fuses four hydrogen nuclei (protons) into one helium nucleus (an alpha particle). This process has many steps, and can take many forms. Below I'll describe common pathways for the fusion reactions.

The proton-proton chain is the most common fusion sequence in most stars. The total reaction is:

$$4\,{}^1_1\mathrm{H} \rightarrow {}^4_2\,\mathrm{He} + 2e^+ + 2\nu_e + 2\gamma \tag{11}$$

For one branch of the P-P chain the reactions are:

$$ {}^1_1\mathrm{H} + {}^1_1\,\mathrm{H} \rightarrow {}^2_1\,\mathrm{H} + e^+ + \nu_e \tag{12}$$

$$ {}^2_1\mathrm{H} + {}^1_1\,\mathrm{H} \rightarrow {}^3_2\,\mathrm{He} + \gamma \tag{13}$$

$$ {}^3_2\mathrm{He} + {}^3_2\,\mathrm{He} \rightarrow {}^4_2\,\mathrm{He} + 2\,{}^1_1\mathrm{H} \tag{14}$$

The first reaction is the slowest. This set of three equations is known as PPI, the most common reaction to fuse hydrogen into helium. There are., however,

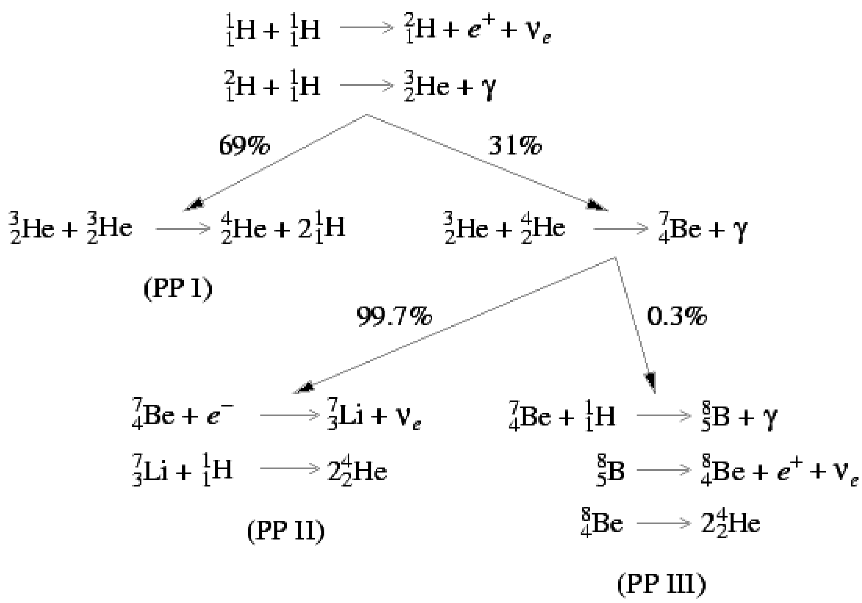

Figure 2: The proton-proton chain. Percentages give the probability for the conditions within the Sun.

other ways to fuse H into He. PPII:

$$^3_2\mathrm{He} + ^4_2\mathrm{He} \rightarrow ^7_4\mathrm{Be} + \gamma \tag{15}$$

$$^7_4\mathrm{Be} + e^- \rightarrow ^7_3\mathrm{Li} + \nu_e \tag{16}$$

$$^7_3\mathrm{Li} + ^1_1\mathrm{H} \rightarrow 2\,^4_2\mathrm{He} \tag{17}$$

PPIII:

$$^7_4\mathrm{Be} + ^1_1\mathrm{H} \rightarrow ^8_5\mathrm{B} + \gamma \tag{18}$$

$$^8_5\mathrm{B} \rightarrow ^8_4\mathrm{Be} + e^+ + \nu_e \tag{19}$$

$$^8_4\mathrm{Be} \rightarrow 2\,^4_2\mathrm{He} \tag{20}$$

Note the production of neutrinos (ν_e)! These will be important later.

The energy generated through the PP chain scales as temperature to the fourth power, when the temperature is around 1.5×10^7 K.

$$q_{PP} \simeq 1.05 \times 10^{-5} \rho X^2 T_6^4 \exp \text{ erg g}^{-1} \text{ s}^{-1} \tag{21}$$

where T_6 is the temperature in units of 10^6 K and ρ is in g cm^{-3}.

CNO Cycle

Larger stars can use the CNO cycle, in addition to PP. CNO stands for carbon, nitrogen and oxygen, but *these are just catalysts!* No C,N, or O are produced

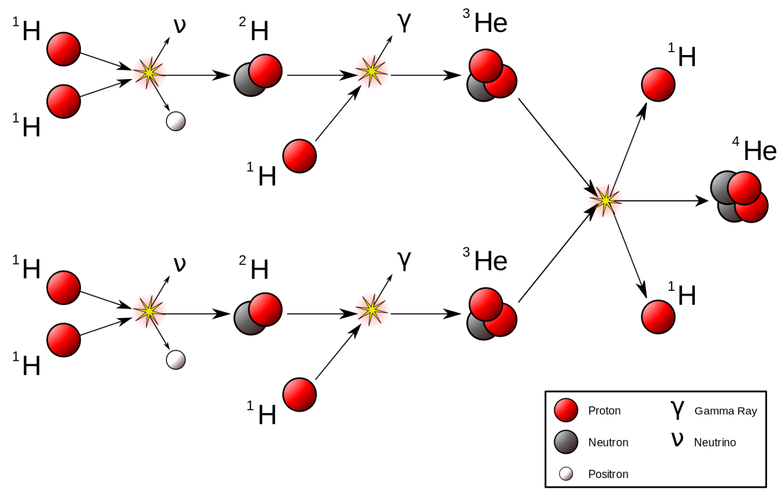

Figure 3: The PPI chain.

in the CNO cycle. Hydrogen is still being converted into helium, just through a different method.

The CNO cycle is *strongly* temperature dependent, the emitted energy scales as temperature to the 19.9 power! This means that at higher core temperatures the CNO cycle takes over energy production.

$$q_{CNO} \simeq 8.24 \times 10^{-24} \rho X X_{CNO} T_6^{19.9} \text{ erg g}^{-1} \text{ s}^{-1} \tag{22}$$

where T_6 is the temperature in units of 10^6 K and ρ is in g cm^{-3}.

PP vs CNO

PP and CNO are the main reactions that power stars when they are on the main sequence.

The reactions of the CNO cycle require more kinetic energy to overcome the Coulomb barrier. Additionally, the CNO cycle's temperature dependence is much stronger. As a result, small stars get essentially all their energy from PP, whereas large stars get essentially all of theirs from CNO. Stellar mass stars get about half their energy from each.

Other Fusion Processes

While on the main sequence, stars fuse hydrogen into helium. Eventually, as we'll see, the hydrogen is exhausted in their cores and they use other fusion processes. We'll discuss stellar evolution in detail, but for now let's talk about the fusion reactions.

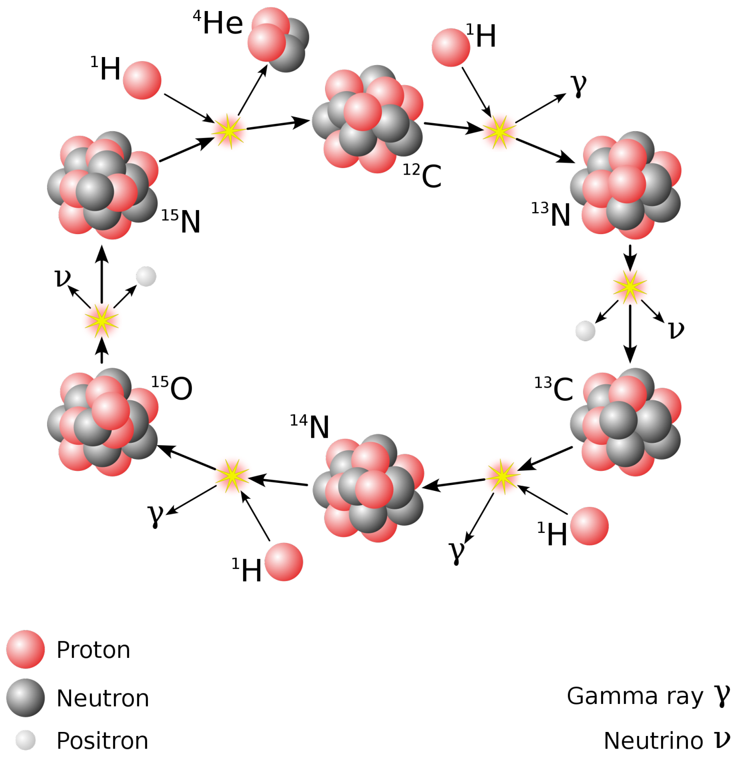

Figure 4: The CNO cycle.

Triple Alpha

At high temperatures, an additional fusion process is available. We learned that there are no stable nuclei with $A = 8$, and although that is the case, ^{8}Be has a lifetime of $2.6 \times 10^{-16}\,s$:

$$^4_2\text{He} + ^4_2\text{He} = ^8_4\text{Be} \tag{23}$$

What temperature is required for alpha particle capture of Be before it decays? Around 10^8 K, which is higher than that of stars on the main sequence.

$$^8_4\text{Be} + ^4_2\text{He} = ^{12}_6\text{C} + \gamma \tag{24}$$

The "triple alpha" process combines three helium nuclei (alpha particles) to create carbon. This reaction produces just 10% of the energy per unit mass of hydrogen fusion. It has an unbelievable temperature dependence:

$$q_{3\alpha} \simeq 4 \times 10^{-8} \rho^2 Y^3 T_8^{41.0} \text{ erg g}^{-1}\text{ s}^{-1} \tag{25}$$

Note the ρ^2! A related process at higher temperatures produces oxygen:

$$^{12}_6\text{C} + ^4_2\text{He} = ^{16}_8\text{O} + \gamma \tag{26}$$

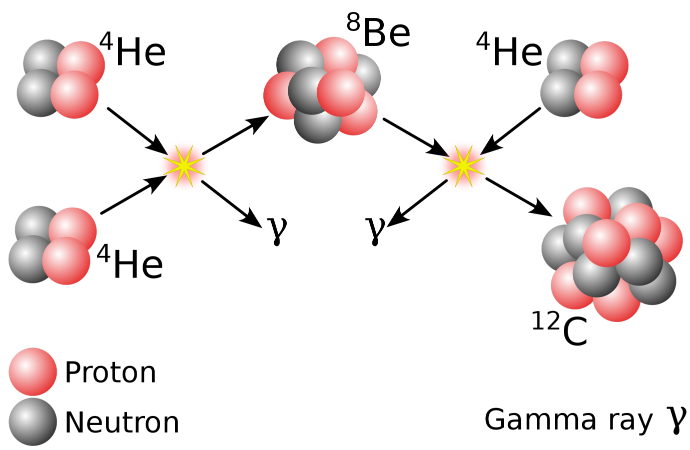

Figure 5: Triple-alpha process.

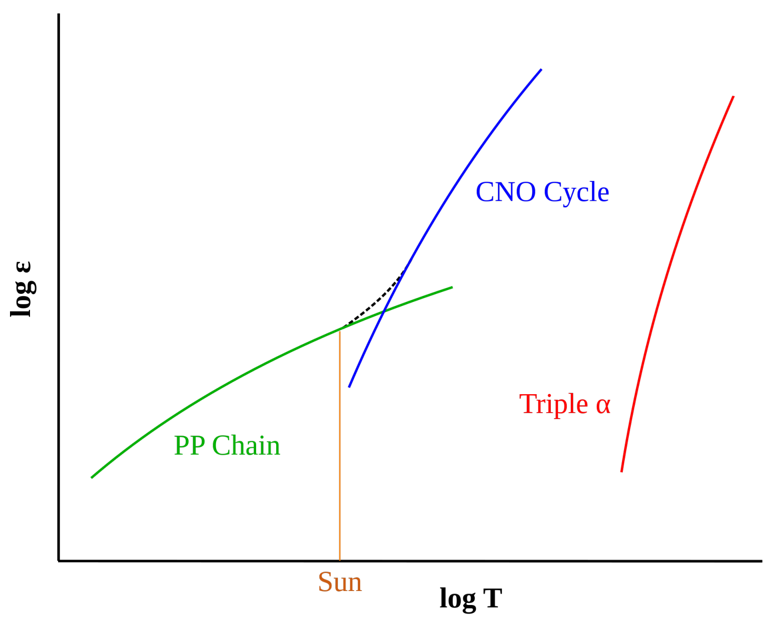

Figure 6: Nuclear energy generation.

Nuclear Fuel	Process	$T_{threshold}$ 10^6K	Products	Energy per nucleon (Mev)
H	PP	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C,O	0.61
C	C+C	600	O,Ne,Ma,Mg	0.54
O	O+O	1000	Mg,S,P,Si	~0.3
Si	Nuc eq.	3000	Co,Fe,Ni	<0.18

Figure 7: Fusion processes

More Fusion processes

At higher temperatures, there are yet more processes available. In general, these reactions are reversible, however, so that a photon can disintegrate a nucleus. These photodisintegrations proceed at higher temperatures than the fusion reactions.

The additional reactions fuse C, O, Mg, and Si. The end of the road is Fe, which as we saw is the peak of the binding energy curve per nucleon.

s- and r-processes

If there are sufficient free neutrons, these can be captured to produce new isotopes. Since the neutron does not interact electronically with a nucleus, the Coulomb barrier doesn't matter and the only limiting factor is the availability of free neutrons.

Neutron capture will continue until an unstable isotope is produced that decays via β-decay. In beta decay, an electron or a positron is emitted and a neutron in the core is converted into a proton. This process creates a new element, and that new element can do neutron capture, and the cycle repeats.

There are two names for this process. "r" for rapid - if the neutron capture rates are faster than the decays, and "s" if they are slower.

The r-process takes place in supernovae because that is the only environment with sufficient free neutrons to create a reaction rate faster than the decay. The s-process does operate in stars during the course of their normal lifetimes (but is more common in evolved stars), and therefore produces elements heavier than iron, but at exceedingly low rates. The problem is that there just aren't

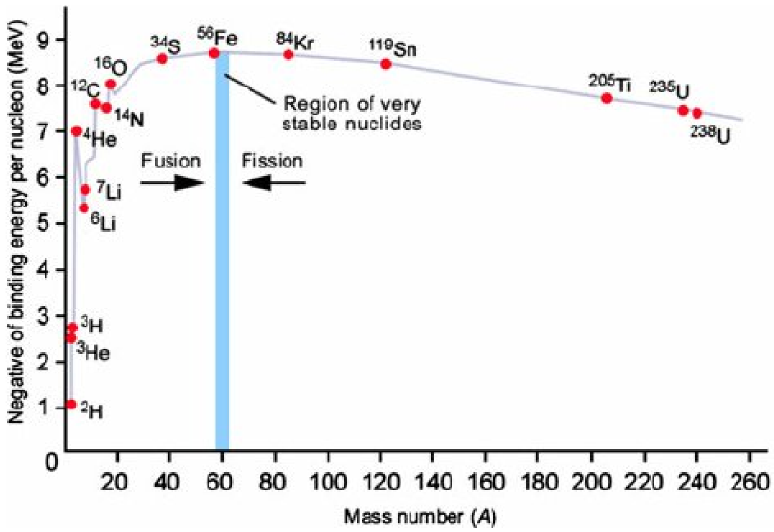

Figure 8: The binding energy curve. Elements to the left of the line release energy via fusion; those to the right release energy via fission.

that many free neutrons. The main neutron source reactions are:

$$^{13}_{6}\text{C} + ^{4}_{2}\text{He} \rightarrow ^{16}_{8}\text{O} + n \tag{27}$$

$$^{22}_{10}\text{Ne} + ^{4}_{2}\text{He} \rightarrow ^{25}_{12}\text{Mg} + n \tag{28}$$

(29)

Neutrinos

How do we know what is going on within a star? Neutrinos! Each nuclear reaction can produce neutrinos. Neutrinos themselves do not like to react with matter, which is good because they stream out of the Sun almost unimpeded, but bad because they are difficult to detect. In our book, section 9.3, there is a nice history of Solar neutrino detectors. Rather than repeat that, I'll summarize with:
- detectors rely on having a large quantity of material (water, gallium, or chlorine)
- incident neutrinos can create reactions in this material
- some materials are more sensitive to low, or high-energy neutrinos, making the detector sensitive to particular fusion reactions.
- Wikipedia lists maybe 50 neutrino experiments
- On average such experiments detect on order 1 neutrino/day.