Stellar - HW5
September 19, 2025, Due September 26, 2025
2 pt each part

1a) At what temperature would the energy generation rates of the PP chain and CNO cycles be equal? Assume Solar abundances. The energy generation rates for these processes are:

$$q_{PP} \simeq 1.05 \times 10^{-5} \rho X^2 T_6^4 \text{ erg g}^{-1} \text{ s}^{-1} \tag{1}$$

and

$$q_{CNO} \simeq 8.24 \times 10^{-24} \rho X X_{CNO} T_6^{19.9} \text{ erg g}^{-1} \text{ s}^{-1} \tag{2}$$

where T_6 is the temperature in units of 10^6 K and ρ is in g cm^{-3}.

b) What are the CNO and P-P chain energy generation rates for the Sun? Use $T = 1.58 \times 10^7$ K, $\rho = 162\,\text{g}\,\text{cm}^{-3}$, $X = 0.34$, and $X_{\text{CNO}} = 0.013$.

2a) Taking into consideration the Maxwell–Boltzmann velocity distribution, what temperature would be required for two protons to collide if quantum mechanical tunnelling is neglected? Assume that nuclei having velocities ten times the root-mean-square (rms) value for the Maxwell–Boltzmann distribution can overcome the Coulomb barrier. Compare your answer with the estimated central temperature of the Sun.

b) Using the M-B distribution below, calculate the ratio of the number of protons having velocities 10× the rms value to those moving at the rms velocity.

$$n_v dv = n\left(\frac{m}{2\pi kT}\right) \exp\left(\frac{-mv^2}{2kT}\right) 4\pi v^2 dv \tag{3}$$

c) Assuming (incorrectly) that the Sun is pure hydrogen, estimate the number of hydrogen nuclei in the Sun. Could there be enough protons moving with a speed ten times the rms value to account for the Sun's luminosity?

3) Lifetimes
a. At what rate is the Sun's mass decreasing due to nuclear reactions? Express your answer in solar masses per year and please state all assumptions.
b. How do the lifetimes of the Sun and a 100 $M_\odot$ star differ if both are able to use 10% of their masses for fusion?

4) (Grad students only) Prove that the energy corresponding to the "Gamow

Peak" is given by

$$E_0 = \left(\frac{bkT}{2}\right)^{2/3} , \tag{4}$$

where

$$b = \frac{\pi\mu_m^{1/2} Z_1 Z_2 e^2}{\pi\epsilon_0 h} , \tag{5}$$

with μ_m the reduced mass. Here, let's use energy forms for the cross section:

$$\sigma(E) = \frac{S(E)}{E} \exp(-bE^{-1/2}) , \tag{6}$$

where $S(E)$ is some slowly varying function, but we can assume it's a constant, and the M-B distribution:

$$f(v)dv = \sqrt{\left(\frac{\mu}{2\pi kT}\right)^3} 4\pi v^2 e^{-\frac{\mu v^2}{2kT}} dv . \tag{7}$$