15M
HW #7

1) for such a relatively high density environment, Case B is better.
The probability is then

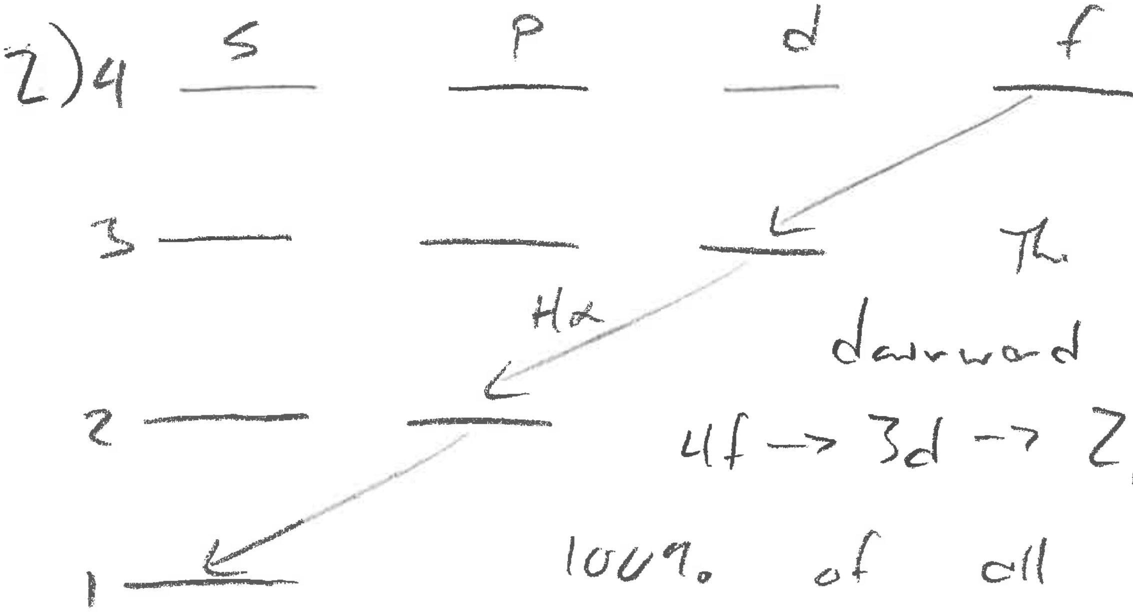
$n_e \alpha_B$ with $\alpha_B = 7.01\times10^{-12}\ cm^3\ s^{-1}$
from Table 14.6

$n_e = 0.01\ cm^{-3} \Rightarrow n_e \alpha_B = 7.01\times10^{-14}\ s^{-1}$

b) now we want α_{rr}, which from Table 14.6 is $\alpha_{rr} = 8.63\times10^{-12}\ cm^3\ s^{-1}$
Recombinations are with electrons, so

$n_e \alpha_{rr} = 0.01 \times 8.63\times10^{-12} = 8.63\times10^{-14}\ s^{-1}$

2)



The only path downward is

$4f \to 3d \to 2p \to 1s$

100% of all 4f recombinations lead to Hα

($\Delta l = 0, \pm 1$ for electric dipole)

b) We can produce Hβ via

$4d \to 2p$, $4p \to 2s$, or $4s \to 2p$.

None of these are accessible from 4f, so 0%

c) We could just add them up, but it's easier to subtract from α_A,

so $\alpha_{>4f} = \alpha_A - \alpha_{\leq 4f}$. The shells fill up as

$$\underbrace{1s\ 2s\ 2p\ 3s\ 3p\ 4s\ 3d\ 4p\ 5s\ 4d\ 5p\ 6s\ 4f}_{\text{add all these } \alpha\text{s}}$$

We don't have α_{5s}, α_{5p}, or α_{6s}, but they are probably small.

Assume $T = 10^4$ K, I get for Case A

$$\alpha_{3d4f} = 1.08 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$$

($\alpha_A = 4.18 \times 10^{-13}$, so ~75% are to levels $< 4f$)

For case B: $\alpha_{3d4f} = 1.07 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$

(these should of course be the same)

3) ${}^2P_{3/2}$ —— $\lambda = 12.8 \mu m \rightarrow h\nu = 1.55 \times 10^{-32}$ J

${}^2P_{1/2}$ —— $E/k =$

From extension of our ionization discussion the collision rate $C \hat{=} \sigma v$. At this point we could use the geometric σ and estimate the velocities from the MB distribution.

We can do better though. Draine eqn. 2.26 says for transitions between states u and ℓ

$$\langle \sigma v \rangle_{u \to \ell} = \frac{8.629 \times 10^{-8}}{\sqrt{T_4}} \frac{\Omega_{u\ell}}{g_u} \ cm^3 \ s^{-1}$$

where $\Omega_{u\ell}$ is the collision strength and g_u is the degeneracy.

$\Omega_{u\ell} = 0.314 \ T_4^{\,0.076 + 0.002 \ln T_4}$ (assume LTE: $\Omega_{u\ell} = \Omega_{\ell u}$)

$g_u = 2J+1 = 4$

$$\Rightarrow \langle \sigma v \rangle_{u \to \ell} \approx \frac{8.629 \times 10^{-8}}{\sqrt{T_4}} \frac{0.314}{4} = \frac{6.8 \times 10^{-9}}{\sqrt{T_4}} \ cm^3 \ s^{-1}$$

$$\langle \sigma v \rangle_{\ell \to u} = 1.4 \times 10^{-8} \ T_4^{-0.5} \ cm^3 \ s^{-1}$$

b) In detailed balance,

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT}$$ [assume high density so ignore A_{lu}]

$g_u = 4 \quad g_\ell = 2$

$$\frac{n_u}{n_\ell} = 2 e^{-1.55\times10^{-22}\,J/kT} \approx 2$$

back to a) for de-excitation,

$\Omega_{u\ell} = \Omega_{\ell u}, \quad g_\ell = 2$

$$\rightarrow \langle \sigma v \rangle_{\ell\to u} = \frac{8.629\times10^{-8}}{\sqrt{T_4}} \frac{0.314}{2}$$

$$= \frac{1.3\times10^{-8}}{\sqrt{T_4}}$$

or $\frac{n_u}{n_\ell} = \frac{\gamma_{\ell u}}{\gamma_{u\ell}} \left(1 + \frac{A_{u\ell}}{n_e \gamma_{u\ell}}\right)^{-1}$

but $A_{u\ell}$ is low.

4) Want $\tau_{13.6eV} = 1$

When $\tau = 1$, $d = mfp$, so $dn\sigma = 1$

From Figure 13.1, $\sigma_{PI} \hat{=} 5 \times 10^{-18}\ cm^2$ at 13.6 eV

$dn = 2 \times 10^{17}\ cm^{-2}$

If $d = 1pc$, $n = 0.07\ cm^{-3}$. If n is greater, use case B.

If $d = 1kpc$, $n = 7 \times 10^{-5}\ cm^{-3}$