ASTR702 - HW9. Last one!
December 1, 2025, Due December 8, 2025
2 pt each part

1) What is the velocity shift of the Sun due to Jupiter?

2) By what fraction does the Sun's flux decrease during Mercury's transit? Mercury orbits at 0.4AU and has a radius of 2.44×10^6 m. Use solid angles!

3) IMF and compact objects

The Kroupa (2001) piecewise IMF is defined as:

$$\xi(m)dm \propto m^{-\alpha}dm \tag{1}$$

where the exponent α depends on mass range:

$$\alpha = \begin{cases} 1.3 & 0.08 < m < 0.5 \\ 2.3 & 0.5 < m < 100\,, \end{cases} \tag{2}$$

where $\xi(m)$ is the number of stars in $m + dm$ and m is in units of the Solar mass.

We need to compute several key integrals for mass ranges relevant to different compact objects. These integrals give the fraction of stars (by count) in specific mass ranges:

$$\int_{0.08}^{0.5} m^{-1.3}\, dm = 4.073 \tag{3}$$

$$\int_{0.5}^{100} m^{-2.3}\, dm = 1.463 \tag{4}$$

$$\int_{8}^{20} m^{-2.3}\, dm = 0.0538 \quad \text{(neutron star progenitors)} \tag{5}$$

$$\int_{20}^{100} m^{-2.3}\, dm = 0.0268 \quad \text{(black hole progenitors)} \tag{6}$$

$$\int_{0.08}^{8} m^{-2.3}\, dm = 1.380 \quad \text{(white dwarf progenitors)} \tag{7}$$

These integrals weight the IMF by mass, giving the total mass per unit time:

$$\int_{0.08}^{100} m \cdot m^{-\alpha(m)}\, dm = \int_{0.08}^{0.5} m^{-0.3}\, dm + \int_{0.5}^{100} m^{-1.3}\, dm \tag{8}$$

$$= 4.073 + 15.97 \tag{9}$$

$$= 20.04 \tag{10}$$

a) Using $\mathrm{SFR} = 2\, M_{\odot}\ \mathrm{yr}^{-1}$, what is the core-collapse SN rate per century?

Hint: the number of core-collapse supernovae produced over a time period T is given by:

$$N_{\mathrm{SN,CC}} = \mathrm{SFR} \times T \times \frac{\int_{m_1}^{m_2} \xi(m)\, dm}{\int_{m_{lo}}^{m_{hi}} m\xi(m)\, dm} \tag{11}$$

- SFR (Star Formation Rate): The rate at which stellar mass is formed, measured in $M_{\odot}$ per year. For the Milky Way, we use $\mathrm{SFR} = 2\, M_{\odot}\ \mathrm{yr}^{-1}$.
- T (Time period): The duration over which we count supernovae, in years.
- $\xi(m)$ (Initial Mass Function): The number of stars born per unit mass interval.
- m_1, m_2 (Mass limits for specific objects): The lower and upper mass limits for the type of object we are calculating.
- m_{lo}, m_{hi} (Overall mass limits): The lower and upper mass limits for all stars in the IMF.
- Numerator $\left(\int_{m_1}^{m_2} \Phi(m)\, dm\right)$: The integral of the IMF over the mass range of interest. This represents the fraction of stars (by count) in that mass range.
- Denominator$\left(\int_{m_{lo}}^{m_{hi}} m\Phi(m)\, dm\right)$: The mass-weighted integral of the IMF over all stars. This normalizes the total stellar mass formed to equal SFR $\times T$.
- Ratio: The ratio of integrals gives the fraction of total stellar mass that goes into objects in the specified mass range. Since each object produces one supernova or becomes one compact object, this directly gives the count.

Over the Galactic age ($T_{\mathrm{gal}} = 10\ \mathrm{Gyr} = 10^{10}\ \mathrm{yr}$), the total population of a particular type of compact object is:

$$N = \mathrm{SFR}_{\mathrm{avg}} \times T_{\mathrm{gal}} \times \frac{\int_{m_1}^{m_2} \Phi(m)\,dm}{\int_{m_{lo}}^{m_{hi}} m\Phi(m)\,dm} \tag{12}$$

Key assumption: All compact objects have effectively infinite lifetimes on Galactic timescales, so this formula gives the cumulative population.

b) How many white dwarfs are in the Milky Way? (Progenitor mass $0.08 < m < 8\,M_\odot$)

c) How many neutron stars are in the Milky Way (Progenitor mass $8 < m < 20\,M_\odot$)

d) How many black holes are in the Milky Way (Progenitor mass $20 < m < 100\,M_\odot$)

4) (Grad students only) Assume you identify a star that is oscillating in position every 50 years due to a transiting (but unseen) companion.

a) If you measure a parallax of $0.4''$, what is the distance to this system?

b) The angular extent of the semimajor axis of the reduced mass is $7.6''$. What is the sum of the masses? Assume that the inclination is 0. Hint: convert angular extent to radians.

c) Now assume you measure the radial velocity of the star in question and find that it is nonzero. What does this imply about your answer in part b)?

d) Assume you measure an apparent visual magnitude for the system of -1 and that you believe the secondary star to be a white dwarf. Solve for the luminosity, then the mass of the primary using the mass-luminosity relationship. The Sun's absolute visual magnitude is +4.8. What is the upper limit to the value of the mass function?