ASTR 702
HW #5

1) $\sigma(E) = \frac{S(E)}{E} e^{-\frac{b}{E^{1/2}}}$

$$f(v)dv = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-\frac{mv^2}{2kT}} dv$$

$$\langle \sigma v \rangle = \int_0^\infty \sigma f(v) v \, dv$$

$$= \int_0^\infty \frac{S(E)}{E} e^{-\frac{b}{E^{1/2}}} \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^3 e^{-\frac{mv^2}{2kT}} dv$$

$$= \underbrace{S(E) \, 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2}} \int_0^\infty \frac{v^3}{E} e^{-(b/E^{1/2} + mv^2/2kT)} dv$$

$E = \frac{1}{2}mv^2$ $\quad dE = mv\,dv \quad dv = \frac{dE}{mv} \quad v^2 = \frac{2E}{m}$

$$\langle \sigma v \rangle = \int_0^\infty \frac{2\cancel{E}}{m} \cdot \frac{v}{\cancel{E}} \cdot \frac{1}{mv} e^{-(b/E^{1/2} + mv^2/2kT)} dE$$

$$= S(E) \frac{8\pi}{m^2} \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty e^{-(b/E^{1/2} + E/kT)} dE$$

$$= \left(\frac{8}{\pi m}\right)^{1/2} \frac{S(E)}{(kT)^{3/2}} \int_0^\infty e^{-(b/E^{1/2} + E/kT)} dE$$

want $\frac{d}{dE} = 0$ so $\frac{\partial}{\partial E}\left[e^{-\left(b/E^{1/2} + E/kT\right)} \right] = 0$

Wolfram alpha gives $E = \left(\frac{bkT}{2}\right)^{2/3}$

2) b) $q_{pp} = 1.05 \times 10^{-5} \rho X^2 T_6^4$

$\rho = 162 \text{ g cm}^{-3}$

$X = 0.34$

$T_6 = 15.8$

$\Rightarrow q_{pp} = 12.2 \text{ erg g}^{-1} \text{ s}^{-1}$

$q_{CNO} = 8.24 \times 10^{-24} \rho X X_{CNO} T_6^{19.9}$

$X_{CNO} = 0.013$

$\Rightarrow q_{CNO} = 4.21 \text{ erg g}^{-1} \text{ s}^{-1}$

2a) $q_{pp} = q_{CNO}$

$$1.05\times10^{-5}\,\cancel{\rho}\,X^{\cancel{2}}\,T_6^{4} = 8.24\times10^{-24}\,\cancel{\rho}\,\cancel{X}\,X_{CNO}\,T_6^{19.9}$$

$$T_6^{15.9} = \frac{1.05\times10^{-5}}{8.24\times10^{-24}}\;\frac{X}{X_{CNO}} = 3.33\times10^{19}$$

$$\Rightarrow T_6 = 16.9$$

$$T = 1.69\times10^{7}\,K$$

3) $\frac{3}{2}kT = \frac{1}{2}\mu v^2$

rms of M-B dist $\quad V_{rms} = \left(\frac{3}{\cancel{2\mu}}\frac{kT}{\mu}\right)^{1/2}$

We need $\frac{1}{2}\mu v^2 > \frac{e^2}{\epsilon_0 \cdot 4\pi r}$ $\cancel{M}$

$$\frac{1}{2}\cancel{\mu}\cdot 10^2 \cdot \frac{3kT}{2\cancel{\mu}} > \frac{e^2}{\epsilon_0 \cdot 4\pi r}$$

$$T > \frac{e^2 \cdot \cancel{4}}{3k\epsilon_0 \cdot 400 \cdot \cancel{4}\pi r}$$

$$> \cancel{\frac{1.67\times10^{-7}}{r}} \qquad \frac{\overset{2.2\times10^{-7}}{\cancel{5.57\times10^{-8}}}}{r}$$

if $r \triangleq 2\times10^{-15}\,m,$ $\quad T > \cancel{2.78\times10^{7}}\,K$

$1.1\times10^{8}\,K$

3b) $$\frac{n_v}{n_{rms}} = \frac{\exp(-mv_{rms}^2/2kT)\,(10\,\cancel{v_{rms}})^2}{\exp(-mv_{rms}^2/2kT)\,(\cancel{v_{rms}})^2}$$

$$= 5.7\times10^{-31}$$

3c) # H atoms

$$\frac{M_\odot}{m_p} = 1.2 \times 10^{57}$$

If temperature everywhere is 1.1×10^8 K

$$N_{react} \simeq \frac{M_\odot}{m_p} \left(\frac{nv}{n_{rms}} \right) \simeq 7 \times 10^{26}$$