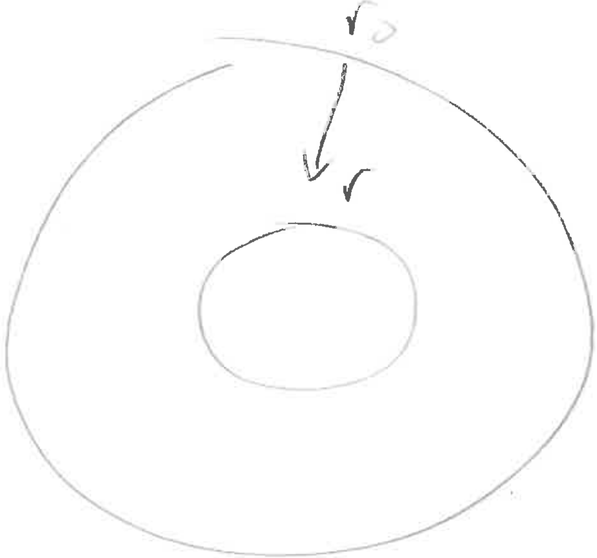
ASTR 702
HW #3

1)



a) $\Omega = GMm\left[\frac{1}{r} - \frac{1}{r_0}\right]$ for test mass

$U = \frac{1}{2}mv^2$

$\frac{1}{2}\cancel{m}v^2 = GM\cancel{m}\left[\frac{1}{r} - \frac{1}{r_0}\right]$

$v^2 = 2GM\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{2GM}{r_0}\left[\frac{r_0}{r} - 1\right]$

$\rho_0 = \frac{3M}{4\pi r_0^3} \Rightarrow v^2 = \frac{8\pi G\rho_0 r_0^3}{3r_0}\left[\frac{r_0}{r} - 1\right]$

$v = \frac{8\pi G\rho_0 r_0^2}{3}\left[\frac{r_0}{r} - 1\right] = \frac{dr}{dt}$

$\Rightarrow t_{ff} = \int_r^0 \left(\frac{3}{8\pi G\rho_0 r_0^2}\right)^{1/2}\left[\frac{r_0}{r} - 1\right]^{-1/2} dr$

Wolfram to the rescue!

$$t_{ff} = \frac{1}{4}\left(\frac{3\pi}{2}\right)^{1/2}\left(\frac{1}{G\rho}\right)^{1/2} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

b) $$\rho_\odot = \frac{M_\odot}{4/3\pi R_\odot^3} = \frac{2\times10^{30}\,kg}{4/3\,\pi\,(7\times10^8\,m)^3} = 1400\ kg/m^3$$

$$\Rightarrow t_{ff,\odot} = 1800\,s = 30\,m$$

c) $$\rho_{WD} = \frac{0.5M_\odot}{4/3\pi R_\oplus^3} = 9.2\times10^8\ kg/m^3$$

$$\Rightarrow t_{ff,WD} = 2.2\,s$$

2a) $f(v) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} \cdot 4\pi v^2 e^{-mv^2/2kT}$ (3D)

$f(v) = \sqrt{\frac{m}{2\pi kT}} e^{-mv^2/2kT}$

A Gaussian!

b) $H\alpha$: $\lambda = 656\,nm = 6.56 \times 10^{-7}\,m$

$T \sim 6000K$

for Gaussian $g(x) \propto e^{-\frac{x^2}{2\sigma^2}}$

so $\frac{1}{2\sigma^2} = \frac{m}{2kT} \Rightarrow \sigma^2 = \frac{kT}{m}$

$\sigma = \left(\frac{kT}{m}\right)^{1/2}$

$= 7000\ m/s$

c) $\frac{n_{HII} n_e}{n_{HI}} = 2\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \frac{g_{HII}}{g_{HI}} e^{-\chi/kT}$

$g_{HII} = 2 \quad g_{HI} = 4$

$\chi = 13.6\,eV = 2.18 \times 10^{-18}\,J$

$$\frac{n_{HII}\, n_e}{n_{HI}} = 1.12\times10^{27}\ m^{-3} \cdot 3.77\times10^{-12}$$

$$= 4.22\times10^{15}\ m^{-3}$$

d) $$\frac{n_1}{n_T} = \frac{g_1\, e^{-E_1/kT_{ex}}}{Z(T_{ex})}$$

$g_1 = 2$

$$Z(T_{ex}) = \sum_{i=1}^{\infty} g_i e^{-E_i/kT_{ex}}$$

$g_i = 2i^2 \qquad E_i = 13.6\,eV\left(1 - 1/i^2\right)$

$$Z(T_{ex}) = \sum_{i=1}^{\infty} 2i^2 e^{-\frac{13.6(1-1/i^2)}{kT}}$$

Assume $T_{ex} = T_k = 6000K$,

$$\frac{n_1}{n_T} = \frac{2}{\sum 2i^2 e^{-\frac{13.6(1-1/i^2)}{kT}}}$$

But need to restrict i since at high i, all are overlapping. Regardless of max i chosen, get $n_1/n_T = 1$ for $T_{ex} = 6000$