ASTR368
HW # 8

1) Infation is the idea that early in its history the Universe expanded very radidly.

It solves the "flatness problem" of $k=0$ or $\Omega=1$, which seems like a strange result since deviations from flatness would be stretched out.

It also solves the "horizon problem" that the Universe is uniform despite not being in causal contact since it was in causal contact before inflation.

2) For black body, $w = 1/3$ and we find $\frac{T}{T_0} = \frac{1}{R} = 1+z$. At recombination,

$z = 1089$, so $\frac{T}{T_0} = 1090$. and $T \sim 3000\,K$

since $T_0 = 2.75\,K$. At $z=1$, $T = 5.50\,K$

and at $z=2$, $T = 8.25\,K$.

3) Many possible ways to show this

$$H^2 = H_0^2 \left[ \Omega_{m,0} R^{-3} + \Omega_{r,0} R^{-4} + \Omega_{\Lambda,0} R^{-3(1+w)} + 1 - \Omega_0 \right]$$

If $w = -1$, then $H^2 \to H_0^2 [\Omega_{\Lambda,0} + 1 - \Omega_0]$ as $R \to \infty$

and expansion rate is constant.

If $w < -1$, then $H^2 \to H_0^2 [\Omega_{\Lambda,0} R^x + 1 - \Omega_0]$ as $R \to \infty$

where $x > 0$ and expansion ever increasing

$\to$ Big Rip

If $w > -1$, $H^2 \to H_0^2 [1 - \Omega_0]$ as $R \to \infty$

$\to$ const. expansion

4) The SN experiments measured m to compute m - M where M is the absolute magnitude of type Ia SN. This measurement gives you a measure of the luminosity distance. If the measured m is higher than expected, the SN are fainter, the luminosity distance is greater, and expansion is accelerating. If SN brighter, distance is less, and expansion is decelerating.

5) $H^2(1-\Omega)R^2 = -kc^2$

$H_0^2(1-\Omega_0) = -kc^2$

$\Rightarrow H^2(1-\Omega)R^2 = H_0^2(1-\Omega_0)$

$$\frac{H}{H_0} = \left(\frac{1-\Omega_0}{1-\Omega}\right)^{1/2}\frac{1}{R} = \left(\frac{1-\Omega_0}{1-\Omega}\right)^{1/2}(1+z)$$

$$H = H_0(1+z)\underbrace{\left[\Omega_{m,0}(1+z) + \Omega_{r,0}(1+z)^2 + \frac{\Omega_{\Lambda,0}}{(1+z)^2} + 1 - \Omega_0\right]}_{Z}^{1/2}$$

$$\frac{H}{H_0} = (1+z)Z^{1/2} = \left(\frac{1-\Omega_0}{1-\Omega}\right)^{1/2}\cancel{(1+z)}$$

$$1-\Omega = \frac{(1-\Omega_0)}{Z}$$

$$\Omega = \frac{\Omega_0 - 1}{Z} + 1$$

$$= \frac{\Omega_0 - 1}{\Omega_{m,0}(1+z) + \Omega_{r,0}(1+z)^2 + \Omega_{\Lambda,0}(1+z)^{-2} + 1 - \Omega_0} + 1$$

b) At $z \to 0$, $\Omega \to \dfrac{\Omega_0 - 1}{\Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} + 1 - \Omega_0} + 1$

Since $\Omega_{\Lambda,0} > \Omega_{m,0} > \Omega_{r,0}$, DE dominates

At $z \to \infty$, $\Omega \to \dfrac{\Omega_0 - 1}{\Omega_{r,0}} + 1$