ASTR705 ISM
HW #2
Due Tuesday, 2/3 in class

1) In preparation for our work on H I, let's use our knowledge of the Boltzmann Equation to determine level populations. For now, all you need to know is that 21 cm H I transitions are between an excited state (level u) and an unexcited state (level ℓ). H I 21 cm emission observations (if optically-thin) measure the amount of H I in the hyperfine excited state. The degeneracies are $g_\ell = 1$ and $g_u = 3$.
(a) (2 pt) What is the expression for $\frac{n_u}{n_\ell}$ in terms of spin temperature?
(b) (1pt) What is the fractional error in the assumption that $n(\mathrm{H\,I}) = (4/3) \times n_{\mathrm{u}}$ if $T_s = 100\,\mathrm{K}$ and
(c) (1 pt) $T_s = 20\,\mathrm{K}$?

2) (3 pt) Derive the following equation for a two-level system in detailed balance:

$$e^{-h\nu/kT_{\mathrm{ex}}} = \frac{e^{-h\nu/kT_k} C_{\mathrm{u}\ell} + \frac{1}{e^{h\nu/kT_R}-1} A_{\mathrm{u}\ell}}{C_{\mathrm{u}\ell} + \left(\frac{1}{e^{h\nu/kT_R}-1} + 1\right) A_{\mathrm{u}\ell}} \tag{1}$$

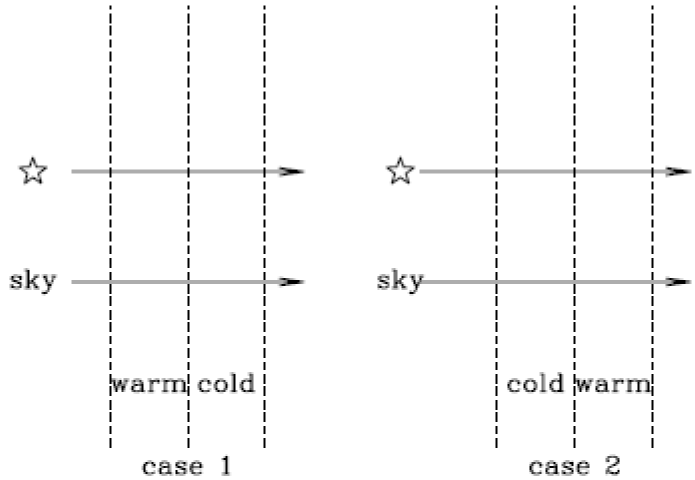
3) Interstellar H I is found with a range of temperatures, but the distribution is bimodal, leading to the concept of two distinct "phases": "cool" H I with spin temperature $T_c \sim 70\,\mathrm{K}$, and "warm" H I with spin temperature $T_w \sim 5000\,\mathrm{K}$. Although we have only just begun our work on H I, we can apply our knowledge of radiative transfer to learn about the expected emission. Suppose that we observe an extragalactic radio source through Galactic H I consisting of a mixture of the cool and warm phases, with spin temperatures of 70 and 5000 K.

The background sky brightness is $I_{\nu,BG}$ (the CMB and background diffuse emission). The total background is therefore the sum of the source and diffuse background components.

We can solve things in terms of intensity or flux. For flux, let Ω be the beamsize of the radio telescope (defined such that a uniform intensity source I_ν gives a measured flux $I_\nu \Omega$).

Let $I_{\nu,source}$ be the intensity of the background source in the absence of any absorption.

(a) (2 pt) What are the expressions for the specific intensity and flux that the telescope will measure at the position of the source if the H I only has one temperature component (ignore the figure for now)?



Two geometries for the cool and warm H I.

(b) (3 pt) What intensity and flux will the telescope measure for the two situations in the figure? Hint: you will need two temperatures, T_c and T_w and two optical depths τ_c and τ_w.

(c) (3 pt) What are the intensity and flux that the telescope will measure when pointed off the source but still through the slabs (the "sky" pointing in the figure)? $I_{\nu,source}$ is known (by making measurements at frequencies where the H I absorption and emission are negligible), show how to determine $(\tau_c + \tau_w)$ from the observed source intensity or flux and off-source intensity or flux.