ASTR367
Midterm
September 18, 2023

Equations

$$f(v) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-\frac{mv^2}{2kT}} \tag{1}$$

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-E_{\mathrm{ij}}/k\,T_{\mathrm{ex}}} \tag{2}$$

$$\frac{n_{i+1} n_e}{n_i} \simeq 2 \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \frac{g_{i+1}}{g_i} \exp\left[-\frac{\Phi_r}{kT}\right] \tag{3}$$

$$\frac{dP}{dr} = -G\frac{M_r \rho(r)}{r^2} = -\rho g \tag{4}$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r) \tag{5}$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \tag{6}$$

$$\frac{dT}{dr}_{\mathrm{rad}} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2} \tag{7}$$

$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} \tag{8}$$

1. The Sun's luminosity is $L_\odot$, its radius is $R_\odot$, its temperature is $T_\odot$, its distance from the Earth is $d_\odot$, and it has an observed radius of $\theta_\odot$.
   a) (2pt) Assuming the Sun emits as a blackbody, use the Stephan-Boltzmann law to give the **expression** for $R_\odot$.
   b) (1pt) What is the **expression** for $R_\odot$ in terms of $\theta_\odot$ and $d_\odot$?
   c) (2pt) What is the **expression** for the flux of the Sun, given its luminosity $L_\odot$ and distance $d_\odot$?
   d) (2pt) If the Sun has an apparent V-band magnitude of -26, and an absolute V-band magitude of $+4$, what is the distance to the Sun in pc?

2. (2pt) What in the geometry and/or properties of the Sun leads to the formation of absorption lines?

3. (4pt) Why are stellar masses limited to the range $0.08\ M_\odot - \sim 100\ M_\odot$? Your answer should give reasons for the upper and lower limits.

4. (3pt) Using the mass-luminosity relationship, $L/L_\odot = (M/M_\odot)^{3.5}$, how would the lifetime of a star of mass $2\ M_\odot$ compare to that of the Sun? Assume the fusion processes and the fraction of mass available for fusion are identical between the Sun and the $2\ M_\odot$ star. (Can leave answer as expression.)

5. (2pt) Give the **expression** for the mass of a single spherical shell of radius R and thickness dR. Assume the star has some density function $\rho(R)$.

6. (2pt) An optical depth of $\tau = 1$ corresponds to one "mean free path" for a photon (the average distance a photon travels before interacting with a particle). If you can see down to a depth of 100 km at $\tau = 1$, how far can you see at an optical depth of $\tau = 0.5$? Assume that the opacity and mass density are constant with depth.

7. (4pt) Draw an H-R diagram, with accurate axes (endpoints). For the x-axis, use temperature. For the y-axis, use Solar luminosities. Indicate the position of the Sun and draw the main sequence.

8. (1 pt each)
   a) At what approximate mass does the initial mass function peak?
   b) At what approximate mass does the CNO cycle provide more energy than the proton-proton chain?
   c) At what approximate mass do stars become fully convective?
   d) At what approximate radius in the Sun is convection more efficient than radiation?

9. (3pt max) Tell me some information you wish I had asked about that we covered in class. To earn points, you must demonstrate that you know the material; don't just list a topic.