ASTR367 - Star Formation
C+O Chapter 12

Stars have to form somehow, because stars exist. The formation of stars must involve collapse of a molecular cloud. These clouds have mean densities of $\sim 10^3$ cm^{-3} or so, and sizes of about a parsec ($\sim 10^{18}$ cm). They must collapse down to $\sim 10^{11}$ cm (the size of a star), with densities of $\sim 10^{24}$ cm^{-3} (the mean density of a star).

When does this collapse occur? When gravity overcomes pressure. The condition where gravity and pressure are in balance is of course called "hydrostatic equilibrium." One treatment says that a cloud not in hydrostatic equilibrium that will collapse has a characteristic size of the "Jeans radius" and mass of the "Jeans Mass" (the condition of instability is the "Jeans Instability"). We will derive these quantities first from the hydrostatic equilibrium condition. We will also do the same derivation using the Virial theorem.

Jeans Mass from Hydrostatic Equilibrium [Following Wikipedia page]

The Jeans mass is named after the British physicist Sir James Jeans, who considered the process of gravitational collapse within a gaseous cloud. He was able to show that, under appropriate conditions, a cloud, or part of one, would become unstable and begin to collapse when it lacked sufficient gaseous pressure support to balance the force of gravity. The cloud is stable for sufficiently small mass (at a given temperature and radius), but once this critical mass is exceeded, it will begin a process of runaway contraction until some other force can impede the collapse. He derived a formula for calculating this critical mass as a function of its density and temperature. The greater the mass of the cloud, the smaller its size, and the colder its temperature, the less stable it will be against gravitational collapse.

Hydrostatic equilibrium is:

$$\frac{dP}{dr} = -\frac{G\rho(r)M_r}{r^2}, \tag{1}$$

where M_r is the enclosed mass, P is the pressure, $\rho(r)$ is the density of the gas at r, G is the gravitational constant and r is the radius. The equilibrium is stable if small perturbations are damped and unstable if they are amplified. In general, the cloud is unstable if it is either very massive at a given temperature or very cool at a given mass for gravity to overcome the gas pressure.

Let's say we have a spherical molecular cloud cloud of radius R, mass M, and sound speed c_s. Compression of this region can only proceed at approximately the sound speed, which gives a characteristic time of:

$$t_{\text{sound}} = \frac{R}{c_s} \tag{2}$$

for sound waves to cross the region. Gravity will attempt to contract the system even further, and will do so on a free-fall time,

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32G\rho}}\,. \tag{3}$$

(This is sometimes given as $t_{\rm ff} = \sqrt{\frac{3}{2\pi G\rho}}$, from a simpler treatment. $t \approx (G\rho)^{-1/2}$ is the characteristic time for many processes in astrophysics. This is a good starting guess for many time scales.) We have collapse when $t_{\rm ff} < t_{\rm sound}$. In this case, the collapse is fast enough that the cloud cannot re-establish equilibrium, which takes place over the timescale given by the sound speed.

It is worth taking a slight detour here to describe how long these free fall times are. For large scales, the growth time for the Jeans instability is

$$\tau_J \simeq 2.3 \times 10^4 \text{yr} \left(\frac{10^6 \text{ cm}^{-3}}{n_H}\right)^{1/2} \tag{4}$$

For $n_H = 1000$ cm^{-3}, this is about 0.7 Myr. Free fall time (collapse timescale for a pressure-less gas) is:

$$\tau_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} = 4.4 \times 10^4 \text{ yr} \left(\frac{10^6 \text{ cm}^{-3}}{n_H}\right)^{1/2} \tag{5}$$

For $n_H = 1000$ cm^{-3} this is 1.4 Myr - slightly longer than growth time.

OK, back to the Jeans mass and radius. The resultant Jeans radius R_J is therefore:

$$\lambda_J \simeq \frac{c_s}{\sqrt{G\rho}} \tag{6}$$

The speed of sound is

$$c_s = \sqrt{\frac{\gamma P}{\rho}}, \tag{7}$$

where γ is the adiabatic index, which is 7/5 for molecular gas and 5/3 for monotonic gas. The pressure $P = nkT = \rho/\mu kT$ assuming an ideal gas, with mean molecular mass μ, so we have

$$R_J \simeq \sqrt{\frac{kT}{G\mu\rho}}\,. \tag{8}$$

The real definition gives a factor of order unity out front:

$$R_J \simeq \sqrt{\frac{15kT}{4\pi G\mu\rho}} \simeq (0.4 \text{ pc}) \left(\frac{c_s}{0.2 \text{ km s}^{-1}}\right) \left(\frac{n}{10^3 \text{ cm}^{-3}}\right)^{-1/2}. \tag{9}$$

All scales larger than the Jeans length are unstable to gravitational collapse, whereas smaller scales are stable.

Perhaps the easiest way to conceptualize Jeans Length is in terms of a close approximation, in which we rephrase ρ as M/r^3. The formula for Jeans' Length then becomes:

$$R_J \approx \sqrt{\frac{k_B T r^3}{G M \mu}}\,, \tag{10}$$

and therefore $R_J = r$ when $kT = \frac{GM\mu}{r}$. In other words, the cloud's radius is the Jeans Length when thermal energy per particle equals gravitational work per particle. At this critical length the cloud neither expands nor contracts. It is only when thermal energy is not equal to gravitational work that the cloud either expands and cools or contracts and warms, a process that continues until equilibrium is reached.

We can recast this in terms of the "Jeans mass":

$$M_J = \left(\frac{4\pi}{3}\right)\rho R_J^3 = \left(\frac{5kT}{G\mu}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2} \simeq (2\ \mathrm{M}_\odot)\left(\frac{c_s}{0.2\ \mathrm{km\ s^{-1}}}\right)^3\left(\frac{n}{10^3\ \mathrm{cm^{-3}}}\right)^{-1/2}. \tag{11}$$

The Jeans mass M_J is just the mass contained in a sphere of radius R_J. It is useful to remember that $M_J \propto T^{3/2}\rho^{-1/2}$. Thus, stars can form most efficiently (when mass is low) in low temperature, high density locations where the Jeans mass is not as great.

The above is an illustrative and wrong derivation! Jeans assumed that the collapsing region of the cloud was surrounded by an infinite, static medium. The pressure in hydrostatic equilibrium is therefore less than that required, and the mass is therefore too high. We will fix this problem below.

A larger issue is that because all scales greater than the Jeans length are also unstable to collapse, any initially static medium surrounding a collapsing region will in fact also be collapsing. As a result, the growth rate of the gravitational instability relative to the density of the collapsing background is slower than that predicted by Jeans' original analysis. This flaw has come to be known as the "Jeans swindle".

The Jeans Mass from the Virial Theorem

We haven't discussed the Virial Theorum yet, but it is covered in your book, Chapter 2. The Virial Theorum says that for *gravitationally bound* systems in equilibrium, the total energy is one-half of the time-averaged potential energy. The gravitiationally bound aspect is important. If the system is not gravitationally bound, the Virial Theorem will not hold. Such cases give rise to contraction or expansion.

We can also derive the Jeans mass using the Virial theorem. Like the condition of hydrostatic equilibrium, the Virial theorem describes a system in equilibrium. If the kinetic energy of a system is K and the gravitational potential energy is U, the simplest incarnation of the Virial theorem says that $2K + U = 0$. An expanding gas cloud will have more kinetic energy than needed ($2K > -U$) and a contracting cloud will have more gravity ($2K < -U$).

Each particle in a gas cloud has kinetic energy, $E = 3/2kT$, so the total kinetic energy $K = \Sigma_i^N E_i = 3/2NkT$, where N is the number of particles. Easy!

The gravitational potential energy in spherical shell of mass dm is

$$dU = -G\frac{M_r dm}{r} . \tag{12}$$

From mass conservation, we know

$$dm = 4\pi r^2 \rho dr . \tag{13}$$

Putting these two expressions together, we find

$$dU = M_r 4\pi r \rho dr . \tag{14}$$

We can integrate this from 0 to R to get the gravitational potential, but not if we don't know how ρ and M_r depend on r. For a constant density sphere,

$$\rho \simeq \bar{\rho} = \frac{M}{4/3\pi R^3} \tag{15}$$

so

$$M_r \simeq 4/3\pi r^3 \bar{\rho} . \tag{16}$$

The integral then gives

$$U \simeq -\frac{3}{5}\frac{GM^2}{R} , \tag{17}$$

which is the gravitational potential for an "isothermal" sphere.

Therefore,

$$3NkT = -\frac{3}{5}\frac{GM^2}{R} . \tag{18}$$

We can replace $N = M/\mu$, with μ the mass per particle, and $R = (3M/4\pi\rho)^{1/3}$ to get

$$M_J = \left(\frac{5kT}{G\mu}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} \tag{19}$$

The same as before!

Fragmentation

Of course this is a simplification – a single cloud does not collapse down to $r = 0$. What happens to complicate the collapse? As the cloud collapse, density rises. Since the collapse is isothermal, a rising density means the Jeans mass of the cloud is falling, so small pieces of the cloud start to collapse on their own. A rising density also means a declining free fall time, so these small dense clumps collapse faster than the overall cloud.

Instead of one giant cloud undergoing a monolithic collapse, the cloud fragments into small collapsing pieces. So what stops this fragmentation? As the density rises, the opacity rises. At some point during the collapse and fragmentation process, the opacity rises high enough that the energy created during the collapse is absorbed within the star itself – it begins to heat up. Since the energy is not lost from the cloud, we call this an adiabatic collapse. Higher temperature means higher pressures (the ideal gas law), which halt the free collapse of the star. Since the cloud absorbs all the gravitational energy of collapse, it heats up, and it starts to act like a blackbody.

At what mass does this happen? We can balance the rate of energy loss through gravitational collapse to the rate at which the cloud radiates blackbody energy, and, solving for the mass, we find $M \approx M_\odot$. In other words, collapse halts when the fragment masses reach star-like masses.

Bonner-Ebert Spheres

In a more realistic scenario, the density is centrally peaked. In this case, the gravitational energy is

$$U = -\frac{3}{5} a \frac{GM^2}{R} \,, \tag{20}$$

where $a > 1$ for centrally peaked density profiles. Mouschovias & Spitzer (1976) find $a \approx 1.67$ for numerical models of clouds on the verge of collapse.

In our above consideration of the Virial theorem, we neglected external pressure and magnetic energy. If we consider the former, with the above modification to the gravitational potential, we arrive at the "Bonner-Ebert mass" (Bonner 1956; Ebert 1957):

$$M_{\rm BE}(p_0) = \frac{225}{32\sqrt{5}\pi} \frac{c_s^4}{(aG)^{3/2}} \frac{1}{\sqrt{p_0}} = 0.26 \left(\frac{T}{10\ \rm K}\right)^2 \left(\frac{10^6\ \rm cm^{-3}\ K}{p_0/k}\right)^{1/2} M_\odot \tag{21}$$

Remember how we said that the Jeans mass neglected some rather important things? Well, the Bonner-Ebert mass is basically the same as the Jeans mass, $M_{\rm BE} \approx 1.18 M_J$. The 18% change is due to the fact that the cloud itself affects the hydrostatic equilibrium assumption before.

Given the typical temperature and pressures of molecular clouds, the Bonner-Ebert mass is about a Solar mass, so it is probably no surprise that this is the peak of the IMF.

Of course we still neglect the magnetic fields. The magnetic energies are similar to the kinetic energies, and so can contribute to the pressure.

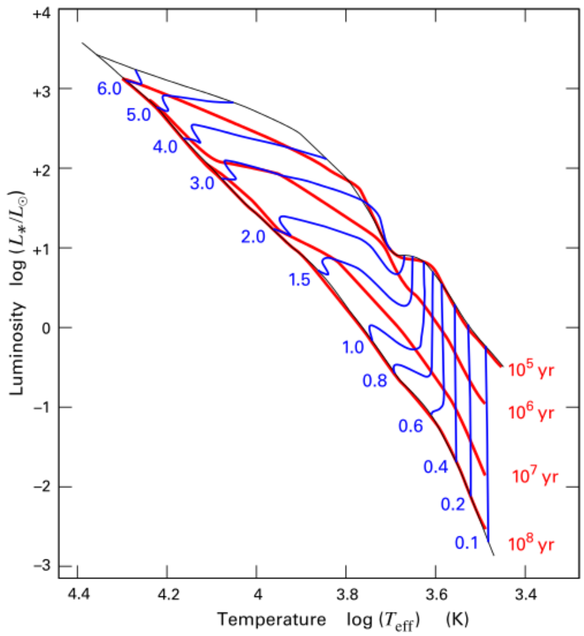

Figure 1: Pre-main sequence evolution

Formation of Actual Stars

During the formation of stars, cores more or less free-fall collapse. The free fall time depends inversely on the density, so the central part collapses first, then the outer parts. What would provide resistance? [Pressure of course!] What would pressure be unimportant? [Cooling from molecular lines!]. This "cooling" of course releases energy that we can detect. This energy peaks in the sub-millimeter to far-infrared. How much energy is liberated? Where does it go?

Pre-Main Sequence Evolution

With a protostar still getting its energy from gravitational collapse, the opacity in the outer layers increases, due to the H^- ion. This causes the protostar to become convective and to lose luminosity. This phase is known as a "Hayashi track."

Eventually, the protostars begin fusion, and are officially stars. EXCEPT for "brown dwarfs," which never reach high enough temperatures for fusion. This limit is at a mass of 0.072 $M_\odot$. Together with the Eddington limit, this sets the mass range that stars can have.

Once star begin fusion, they are on the main sequence. When they first reach the main sequence, they are on what is called the "zero-age main sequence." Stars continue to evolve a bit on the main sequence, and the zero-age main sequence is their starting point.