ASTR705 ISM
Final Exam
Due May 4, 2020, by 5pm, preferably in my box or under my door
63 pt total
Please hand in the problem sheet as well.

1 HI and Radiative Transfer (14 pts)

1) (6 pts, 2 pts each) The general equation for the absorption coefficient is:

$$\kappa_\nu = n_\ell \frac{g_\mathrm{u}}{g_\ell} \frac{A_{\mathrm{u}\ell}}{8\pi} \lambda^2_{\mathrm{u}\ell} \phi_\nu \left[1 - \mathrm{e}^{-h\nu_{\mathrm{u}\ell}/kT_\mathrm{ex}}\right] . \tag{1}$$

a) What is the exponential ($\mathrm{e}^{-h\nu_{\mathrm{u}\ell}/kT_\mathrm{ex}}$) correcting for?

b) Aside from the 21 cm line of H I, for what other lines that we have studied may this term be significant?

c) Why can we use $h\nu_{\mathrm{u}\ell}/kT_\mathrm{ex}$ in place of $\left[1 - \mathrm{e}^{-h\nu_{\mathrm{u}\ell}/kT_\mathrm{ex}}\right]$ for 21 cm H I emission?

2) (8 pts, 4 pts each) Heiles & Trolund derived H I temperatures from observations in the directions of background AGNs. Assume you are observing an AGN in the direction $(\ell, b) = (30\,°, 0\,°)$ that has a 21 cm continuum intensity of 20 K. You observe two positions: one in the direction of the AGN and one just to the side, sampling the same H I gas.

a) If you measure an absorption line with an intensity at line center $\Delta T = T^\mathrm{on} - T^\mathrm{off} = 5$ K and 15 $\mathrm{km\,s^{-1}}$ linewidth in the direction of the AGN, what can you say about the temperature and optical depth of the H I along this line of sight? Please state all assumptions explicitly.

b) Draw the spectrum of H I in the direction of the AGN (on-source) and of H I observed just to the side of the AGN (off-source). Assume the line is centered at the maximum velocity along the line of sight (determine what this is). Put the y-axis in both τ and K.

2 Molecules (10 pts)

3) (10 pts) You have the option to observe a $T \simeq 20$ K, $n \simeq 10^4\,\mathrm{cm^{-3}}$ molecular clump in 4 different molecular lines: ^{12}CO $J = 1-0$, CS $J = 3-2$, NH_3 (1,1), and N_2H^+ $J = 1-0$.

a) (3 pt) Which transitions would have the best chance at detection? In your answer, please include discussion of critical densities and temperatures at which you may expect the transition to occur (ignore the abundance of the molecules themselves).

b) (1 pt) Although the CO and N_2H^+ transitions above are both $J = 1 - 0$, they occur at different frequencies. Why exactly is this the case?

c) (3 pt) For ^{12}CO $J = 1 - 0$, plot (approximately) the CO excitation temperature as a function of density, and label the critical density with a vertical line. What values for $T_{\rm rad}$ and $T_{\rm K}$ did you assume, and why?

d) (3 pt) Would you expect the transitions to have the same line widths? Why or why not? Include a calculation of the thermal line width of CO in your answer.

3 Dust (14 points)

4) (3 pts) If you measure a (background subtracted) dust intensity of 10 Jy beam^{-1} with a telescope that has $5'$ (diameter) resolution, what is the optical depth at 100 μm assuming a temperature of 30 K?

5) (3 pts) By balancing energy absorbed with that emitted, drive an expression for the steady state temperature of a grain exposed to a radiation field with specific energy u_*. Note that $u_* c$ has units of erg s^{-1} cm^{-2}.

6) Extinction! (8 pts, 2 pts each)

a) Shown below in Figure 1 is a color-color diagram from Indebetouw et al. (2005), using 2MASS J, H, and K_s photometric bands. As you did in your homework, derive the color excess ratio E_{H-K_s}/E_{J-K_s} from the fit in this plot (very approximately).

b) Using values in Indebetouw, this ratio can be related to the extinction ratio with:

$$\frac{A_H}{A_K} = \frac{1.5 E_{H-K}}{E_{J-K}} + 1\,, \tag{2}$$

where A_λ is the extinction at wavelength λ. Using your value in part a), what is A_H/A_K (very approximately)?

c) Assume you are observing a particular star with 2 magnitudes of K-band extinction. What is its approximate H-band optical depth (yes, optical depth, not extinction)?

d) For the extinction law (extrapolated from H and K_s bands and assumed to be linear), what is R_V?

4 Ionized Gas and Dynamics (15 points)

7) OIII (5 pt, see Figure 2)

a) (1 pt) The line ratio for the [OIII] transitions at 52 μm ($^3P_2 \rightarrow {}^3P_1$) and 88 μm ($^3P_1 \rightarrow {}^3P_0$) can be used as a density diagnostic. What about these transitions makes them appropriate for constraining density?

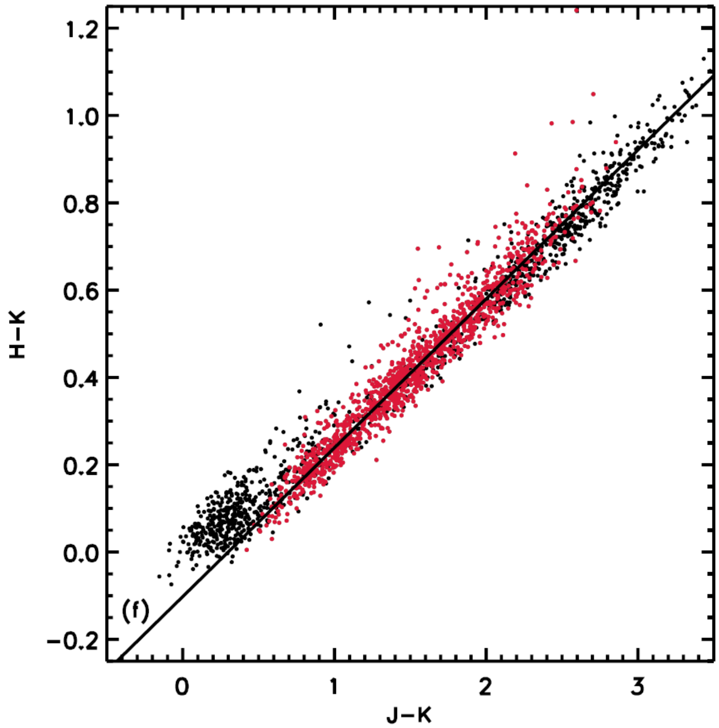

Figure 1: $H - K_s$ versus $J - K_s$ diagram for point sources (Indebetouw et al., 2005).

b) (2 pt) We never see [OIII] lines in a laboratory setting. What does the bracket nomenclature signify, and why do we not see these lines in the lab?

c) (1 pt) The emission line ratio for the 4364Å ($^1S_0 \rightarrow {}^1D_2$) and 5008Å ($^1D_2 \rightarrow {}^3P_2$) [OIII] transitions can be used as a temperature diagnostic. What about these transitions makes them appropriate for constraining temperature?

d) (1 pt) We don't observe the $^3P_2 \rightarrow {}^3P_0$ transition in the ISM - why not?

8) (6 pts) You are observing a pulsar through an H II region. Your observations show a dispersion measure of 200 cm^{-3} pc.

a) (3 pts) If the H II region is $5'$ in radius at a distance of 3 kpc, what is $\langle n_e \rangle$?

b) (3 pts) We derived on a homework an equation for the radio recombination line strength, which is in "Tools of Radio Astronomy" as:

$$\tau = 1.92 \times 10^3 \left(\frac{T_e}{\mathrm{K}}\right)^{-5/2} \left(\frac{\mathrm{EM}}{\mathrm{cm^{-6}\,pc}}\right) \left(\frac{\Delta\nu}{\mathrm{kHz}}\right)^{-1}, \tag{3}$$

where $\Delta\nu$ is the FWHM line width in kHz. Use this equation to derive the intensity (not optical depth) in mK at 5 GHz. Assume a reasonable electron temperature and equal thermal and turbulent broadening.

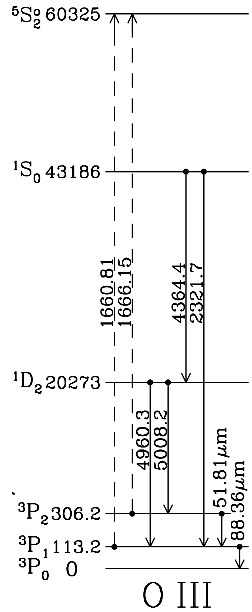

Figure 2: The energy level diagram of OIII.

9) (4 pts, 2 pts each) A supernova goes off, injecting energy into the ISM.

a) If we assume that the mass of the ejecta is 4 $M_\odot$ and the initial velocity is 5000 $\mathrm{km\,s^{-1}}$, conserve momentum (easiest not using the jump conditions!) to derive the mass swept up by the supernova when the velocity is 10 $\mathrm{km\,s^{-1}}$.

b) If the shock wave is propagating in the CNM, show that the shock becomes transonic ($M \simeq 1$) at 10 $\mathrm{km\,s^{-1}}$ (ignore magnetic fields). For reference, assume that the sound speed in the WIM is 1 $\mathrm{km\,s^{-1}}$.

5 Other (10 pt)

10) (2 pt) How can a hydrogen atom depopulate its $2s$ state?

11) (3 pts) The lowest energy vibrational transition of H_2 $(0-0\,\mathrm{S}(0))$ is at $28\,\mu$m. Estimate the fraction of H_2 molecules in the molecular ISM (in LTE) that have been collisionally excited to produce the $0-0\,\mathrm{S}(0)$ line. The degeneracies are $g_J = 2J+1$.

12) (5 pts) The figure below has five emission components indicated with dashed lines. They sum to the solid line, which represents the interstellar radiation field in our Galaxy along a particular sight line. Please label the emission mechanism for each component by drawing on this figure (on the exam itself).

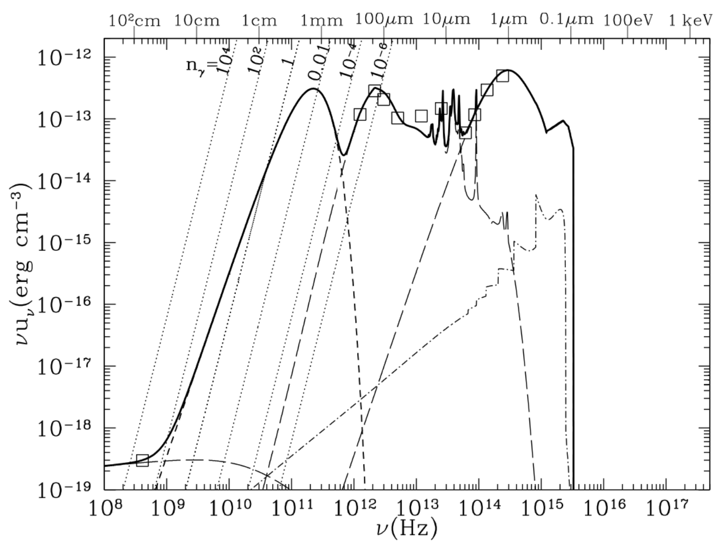

Figure 3: Interstellar radiation field for Question 12.