Stellar - HW4
September 12, 2025, Due September 19, 2025
2 pt each part

1) At what density is the degenerate pressure greater than the non-degenerate pressure? What radius is this in a star like the Sun?

2) Do the same calculation for when the radiation pressure is greater than the gas pressure.

3) At this point in the semester, we have derived the equations of stellar structure (we'll get to the luminosity equation soon if we haven't already). Although we are lacking in our knowledge of the physics of some terms, we can use this knowledge to create a very basic model star.

You will be computing stellar properties as a function of radius. You will submit the spreadsheet or code over email, and write out any relevant expressions you use on the turned-in assignment. Graphs can be printed out or included in the spreadsheet.

Our four equations are:
1) mass conservation,

$$m(r) = \int_0^r 4\pi r'^2 \rho(r')\, dr', \tag{1}$$

2) hydrostatic equilibrium,

$$\frac{dP}{dr} = -\frac{G\, m(r)\, \rho(r)}{r^2}, \tag{2}$$

3) the "luminosity equation":

$$\frac{dL}{dr} = 4\pi r^2 \epsilon, \tag{3}$$

where ϵ is the amount of energy produced per unit volume,
4) the temperature gradient equation:

$$\frac{dT}{dr} = \frac{3\kappa\rho L}{64\pi r^2 \sigma T^3}, \tag{4}$$

where κ is the opacity and
5) the ideal gas law:

$$P(r) = \frac{\rho(r) k_B T(r)}{\mu m_{\rm p}} \tag{5}$$

We do not yet have expressions for ϵ and κ, so let's ignore the luminosity and temperature gradient equations for now.

We will use an analytic density profile

$$\rho(r) = \rho_c\left(1 - \frac{r}{R_\odot}\right)^\alpha, \qquad \alpha = 3. \tag{6}$$

and derive the temperature profile the ideal-gas law (gives $T(P,\rho)$) and hydrostatic equilibrium. Use a Sun-like radius $R_\odot$ and central density ρ_c.

Suggested numerical procedure:

1. Choose a radial grid for r. I suggest at least 100 steps of r, but it's up to you.

2. Compute $\rho(r_i)$.

3. Compute $m(r_i)$ using the trapezoid rule, the midpoint rule, or the exact analytic solution. Let me know if you want a hint.

4. Integrate the hydrostatic equation *inward* from the surface, using the boundary condition $P(R_\odot) = 0$. The inward finite-difference step (backward integration) is
$$P_{i-1} = P_i + \frac{G\,m_i\,\rho_i}{r_i^2}\,\Delta r,$$
where indices decrease as you step toward the center ($i = N, N-1, \ldots, 1$). Integrating inward avoids guessing a central pressure P_c.

5. Compute $T(r_i)$ from the ideal-gas law.

Deliverables:

- Submit your spreadsheet or code (by e-mail) and write the key equations you used on the turned-in assignment.

- Provide plots of $T(r)$, $\rho(r)$, $m(r)$, $P(r)$, and $\dfrac{d\ln P}{d\ln T}$ as functions of radius r.

- For each plot, briefly compare the trends to those shown in the textbook and comment on whether your simple analytic density produces realistic central values.

- (Grad students only) How do things change with $\alpha = 4$?