ASTR367 - HW9
November 10, 2023, Due November 17, 2023
2 pts each question part, unless noted

1) Kroupa (2001) modeled the Initial Mass Function (IMF) as

$$\xi(m) \propto m^{-\alpha_i} = m^{\gamma_i},$$

where

$$\alpha_0 = +0.3 \pm 0.7, \quad 0.01 \leq m/\mathrm{M}_\odot < 0.08,$$
$$\alpha_1 = +1.3 \pm 0.5, \quad 0.08 \leq m/\mathrm{M}_\odot < 0.50,$$
$$\alpha_2 = +2.3 \pm 0.3, \quad 0.50 \leq m/\mathrm{M}_\odot < 1.00,$$
$$\alpha_3 = +2.3 \pm 0.7, \quad 1.00 \leq m/\mathrm{M}_\odot,$$

where $\xi(m)$ is the number of sources within the mass range $m + \Delta m$. The IMF is defined per volume - let's take the volume as that of the entire Milky Way.

To get the number of sources within a mass range, you can integrate: $N = \int \xi(m)dm = A\int m^{-\alpha_i}dm$, where A is a constant that sets the total number.

a) Plot this IMF (with arbitrary y-axis values). A sketch is fine. You can either plot $\xi(m)$ or N, which of course will differ by an exponent of 1.

b) Can you determine the leading constants for each mass range assuming there are 10^{12} stars that have been created in the history of the Milky Way? Make sure that the edges of the broken power law line up.

c) How many brown dwarfs have been created in the history of the Galaxy?

2a) Use Kepler's 3rd Law to compute and plot the velocities of the eight planets of the solar system compare as a function of their distance. The distances are 0.4, 0.7, 1.0, 1.5, 5.2, 9.6, 19.2, and 30AU for M,V,E,M,J,S,U,N.

b) Assuming they orbit with an eccentricity of zero, can you derive a relationship between the orbital velocity and the orbital distance?

3) For Jupiter, compare the orbital period using the complete version of Kepler's third law to that using $P^2 = a^3$. Do this as a ratio.

4) Use the Vis-viva equation for a parabolic orbit ($a \to \infty$) to derive the escape speed.