Neutron Stars
C+O Chapter 16

Neutron Stars

Stars with masses $> 8-10\ M_\odot$ produce neutron stars instead of white dwarfs. The electron degeneracy that creates white dwarfs is not strong enough to support the increased mass, leading to neutron degeneracy (neutrons are also fermions). The process of creating neutron stars is that of electron capture. At high densities ($\sim 1 \times 10^{10}\,\mathrm{kg\,m^{-3}}$), the low energy arrangement allows for

$$p^+ + e^- \rightarrow n + \nu_e \tag{1}$$

Thus, at high densities, the stellar remnant is entirely neutrons. (Note that the protons are actually within heavy ions, so it's a bit more complicated.)

Neutron stars are supported by neutron degeneracy pressure instead of electron degeneracy pressure. We can use the same arguments as we did for WDs to derive the expression for NSs:

$$R \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{GM^{1/3}} \left[\frac{1}{m_H}\right]^{8/3} . \tag{2}$$

(The derivation of this equation simply replaces the electron mass with the hydrogen (proton) mass.) Evaluating this expression returns $R \approx 3\,\mathrm{km}$. The actual radius of a neutron star is about $10\,\mathrm{km}$.

Just like WDs, neutron stars follow the mass-volume relationship: $MV = constant$. Just like WDs, they also have a mass limit. For NSs, this limit is $2.2\ M_\odot$ or non-rotating and $2.9\ M_\odot$ for rotating. Collapse of a neutron star leads to a black hole.

NS Rotation

Neutron stars are rapidly rotating, due to conservation of angular momentum during collapse. Angular momentum is $L = I\omega$, so we can write

$$I_i\omega_i = I_f\omega_f . \tag{3}$$

The moment of inertia $U = CMR^2$ where C is a constant that depends on the geometry ($C = 2/5$ for a sphere). Thus, assuming the mass is unchanged.

$$R_i^2\omega_i = R_f^2\omega_f . \tag{4}$$

$$\omega_f = \omega_i \left(\frac{R_i}{R_f}\right)^2 . \tag{5}$$

But what is the initial radius? It's the radius of the core, which is basically the radius of a WD. Dividing our two expressions for the radius of NS and WDs, we find

$$\frac{R_{\rm core}}{R_{\rm ns}} \approx 500 \tag{6}$$

Thus, a NS is spinning 10^4 times faster than its progenitor! That's a lot!

Magnetic Fields

NS also have strong magnetic fields, for the same reason. Magnetic flux must be conserved during collapse

$$\Phi = \int_S B \cdot dA\,, \tag{7}$$

where A is the surface area. Assuming a sphere, $A = 4\pi R^2$ and we therefore have

$$B 4\pi R_i^2 = B 4\pi R_f^2 \tag{8}$$

$$B_{ns} = B_i \left(\frac{R_i}{R_{ns}}\right)^2\,, \tag{9}$$

which is the same factor as we had previously. The magnetic field must also increase by $\sim 10^4$ times.

NS Temperatures

NS are hot! They begin their lives at $\sim 10^{11}$ K and like WDs cool slowly as they age. Using this temperature and a radius of $\sim$ 10 km, Stephan-Boltzmann gives $L = 10^{26}$ W, which is similar to that of the Sun. From Wien's law, the peak is near 3 cm, in the X-ray regime.

Pulsars!

Neutron stars emit radiation, and this radiation can be beamed toward us. In such cases, we get one "pulse" of radiation each rotation. Such objects are called "pulsars." Pulsars were first detected by Joclyn Bell, then a grad student, and her advisor Anthony Hewish in 1967. She noticed a "scruff' in her chart record that had a reglar period. She initially thought it may be from aliens, but after finding more sources of scruff, she realized that there were many such pulsars in the sky, and the alien hypothesis was not likely. This discovery led to a nobel prize (for Hewish).

There are a few thousand pulsars known, and the next generation of radio telescopes will undoubtedly find many multiples more (FAST in China is rumored to be finding tons).

Pulsars have periods ranging from seconds to milliseconds. Because they are so regular in their pulses, we can accurately measure their periods, and also period derivatives $\dot{P}$. Pulsar lifetimes can be expressed as $P/\dot{P}$ - a characteristic value is $10^7 - 10^8$ years.

Pulsars generally have periods of between 0.25 and 2 s, with $\dot{P}$ on the order of 10^{-15} s.

How do we know that pulsars are small? Let's balance cetripital acceleration with gravity. If centripital acceleration were larger than the gravitational acceleration, the object would fly apart.

$$\omega^2 R = G\frac{M}{R^2} \tag{10}$$

Since $P = 2\pi\omega$, this expression solves to

$$P = 2\pi\sqrt{\frac{R^2}{GM}}\,. \tag{11}$$

or

$$R = \left(\frac{P^2 GM}{4\pi^2}\right)^{1/3}\,. \tag{12}$$

If we take $P = 1\,s$, I find $R = \simeq 10^4$ km. The fastest pulsars, however, rotate with $P \simeq 0.001$ s, which gives ~ 100 km, which is way too small for a WD.

Three distinct classes of pulsars are currently known to astronomers, according to the source of the power of the electromagnetic radiation:

- rotation-powered pulsars, where the loss of rotational energy of the star provides the power,
- accretion-powered pulsars (accounting for most but not all X-ray pulsars), where the gravitational potential energy of accreted matter is the power source (producing X-rays that are observable from the Earth),
- magnetars, where the decay of an extremely strong magnetic field provides the electromagnetic power.

For rotation-powered pulsars we can calculate the rate of energy loss. The rotational energy is:

$$K = 1/2 I\omega^2 = \frac{2\pi^2 I}{P^2} \tag{13}$$

and therefore energy is lost as

$$\frac{dK}{dt} = -\frac{4\pi^2 I\dot{P}}{P^3} \tag{14}$$

The term $-dK/dt$ is the luminosity. If we assume $P = 1$ s, $\dot{P} = 10^{-15}$ s, and $I = 2/5MR^2 = 2/5 \times 1\ M_{\odot} \times (10\,\text{km})^2$, I get 3×10^{24} W.

Pulsar emission is still poorly understood, although we have plenty of theories. The radiation does imply an increase in spin period though, and a positive value of $\dot{P}$. When a pulsar's

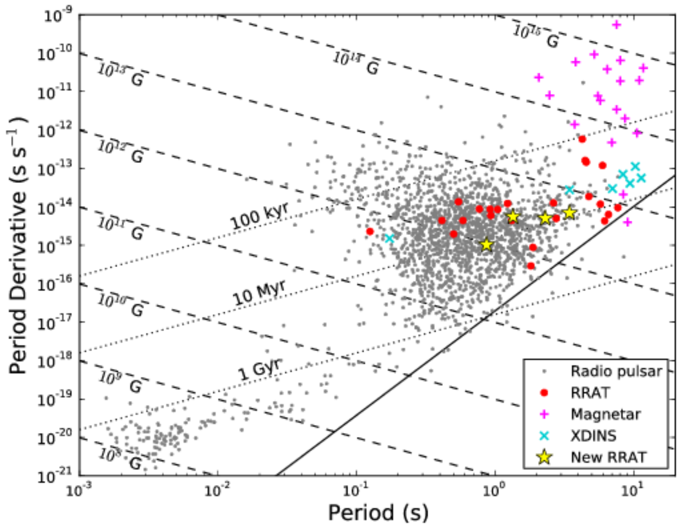

Figure 1: The $P - \dot{P}$ diagram for pulsars.

spin period slows down sufficiently, the radio pulsar mechanism is believed to turn off (the so-called ”death line”). This turn-off seems to take place after about 10-100 million years, which means of all the neutron stars born in the 13.6-billion-year age of the universe, around 99% no longer pulsate.

Given the dispersion relation

$$\omega^2 = k^2c^2 + \omega_p^2. \tag{15}$$

where the angular plasma frequency is

$$\omega_p^2 \equiv \frac{4\pi n_e e^2}{m_e} \tag{16}$$

The plasma frequency is related to the electron density as

$$\nu_p = \frac{\omega_p}{2\pi} \approx 8.979 \text{ kHz } \left(\frac{n_e}{\text{cm}^{-3}}\right)^{1/2}. \tag{17}$$

The propagation speed is given by the group velocity

$$
\begin{aligned}
v_g \equiv \frac{\partial \omega}{\partial k} &= \frac{\partial}{\partial k}\omega && (18)\\
&= \frac{\partial}{\partial k}\left(k^2c^2+\omega_p^2\right)^{1/2} && (19)\\
&= \frac{\cancel{2}kc^2}{\cancel{2}\left(k^2c^2+\omega_p^2\right)^{1/2}} && (20)\\
&= \frac{\cancel{\frac{1}{c}}\left(\omega^2-\omega_p^2\right)^{1/2}c^{\cancel{2}}}{\omega} && (21)\\
&= c\left(1-\frac{\omega_p^2}{\omega^2}\right)^{1/2} && (22)\\
&= c\left(1-\frac{\nu_p^2}{\nu^2}\right)^{1/2} && (23)\\
&\equiv c\mu && (24)
\end{aligned}
$$

where $\mu \leq 1$ is the index of refraction. Below the plasma frequency, μ is imaginary and the waves cannot propagate. For the ionosphere, the electron density peaks at about 10^6 cm^{-3} and so the plasma frequency is about 9 MHz. In the ISM, for $n_e \sim 0.1$ cm^{-3}, the plasma frequency is about 3 kHz.

Dispersive Time Delay

The total propagation time as a function of path length through the medium is

$$
\begin{aligned}
t_{\text{total}} &= \int_0^D \frac{dl}{v_g} && (25)\\
&= \int_0^D \frac{dl}{c}\left(1-\frac{\nu_p^2}{\nu^2}\right)^{-1/2} && (26)\\
&\approx \int_0^D \frac{dl}{c}\left(1+\frac{\nu_p^2}{2\nu^2}\right) && (27)\\
&= \int_0^D \frac{dl}{c} + \int_0^D \frac{dl}{c}\frac{\nu_p^2}{2\nu^2} && (28)\\
&= \frac{D}{c} + \frac{e^2}{2\pi m_e c}\frac{\int_0^D n_e(l)dl}{\nu^2} && (29)\\
&= t_{\text{geometric}} + t_{\text{dispersive}}. && (30)
\end{aligned}
$$

where in the last step we broke up the total time into the geometric travel time and the dispersive delay. Therefore,

$$t_{\text{dispersive}} = \frac{e^2}{2\pi m_e c} \frac{\int_0^D n_e(l)dl}{\nu^2} \tag{31}$$

$$\equiv K \frac{\text{DM}}{\nu^2} \tag{32}$$

$$\approx 4.149 \text{ ms} \left(\frac{\text{DM}}{\text{pc cm}^{-3}}\right)\left(\frac{\nu}{\text{GHz}}\right)^{-2} \tag{33}$$

where $\text{DM} \equiv \int_0^D n_e(l)dl$ is the dispersion measure and $K \equiv \frac{e^2}{2\pi m_e c} \approx 4.149$ ms GHz^2 pc^{-1} cm^3 is the dispersion constant.

Pulsar Utility

Pulsars have proven themselves to be incredibly useful objects for for studying the interstellar medium and for testing concepts in physics.

The radiation from pulsars passes through the interstellar medium (ISM) before reaching Earth. Free electrons in the warm (8000 K), ionized component of the ISM and H II regions affect the radiation in by introducing a frequency-depending delay in the pulse arrival times.

Because of the dispersive nature of the interstellar plasma, lower-frequency radio waves travel through the medium slower than higher-frequency radio waves. The resulting delay in the arrival of pulses at a range of frequencies is directly measurable as the dispersion measure of the pulsar. The dispersion measure is the total column density of free electrons between the observer and the pulsar:

$$DM = \int n_e d\ell \tag{34}$$

where n_e is the electron density of the ISM and the integration is along the path. The dispersion measure is used to construct models of the free electron distribution in the Milky Way.

Pulsars have also been used to detect the so-called stochastic background of gravitational waves. This signal is produced from the combined effects of all supermassive black holes in the Universe. There are 3 consortia around the world which use pulsars to search for gravitational waves. In Europe, there is the European Pulsar Timing Array (EPTA); there is the Parkes Pulsar Timing Array (PPTA) in Australia; and there is the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) in Canada and the US. Together, the consortia form the International Pulsar Timing Array (IPTA). The pulses from Millisecond Pulsars (MSPs) are used as a system of Galactic clocks. Disturbances in the clocks will be measurable at Earth. A disturbance from a passing gravitational wave will have a particular signature across the ensemble of pulsars, and will be thus detected.