Stellar Pulsations
C+O Chapter 14, Prialnik Ch 9

We have discussed stellar pulsations in a few contexts so far. These pulsations give rise to luminosity, and therefore magnitude, fluctuations.

Pulsating stars are important for astronomy. Stellar pulsations reveal physics at work in stellar interiors, and they can even be used to determine distances to stars.

Astronomers have classified a large number of pulsating stars. Although these stars can have different masses and are at different points in their evolutions, we can describe their physics in similar ways.

There are probably several million pulsating stars in the Milky Way (out of $\sim 10^{11}$), or $1 : 10^5$. Many pulsating stars live in the "instability strip" in the H-R diagram.

Types of Variable Stars

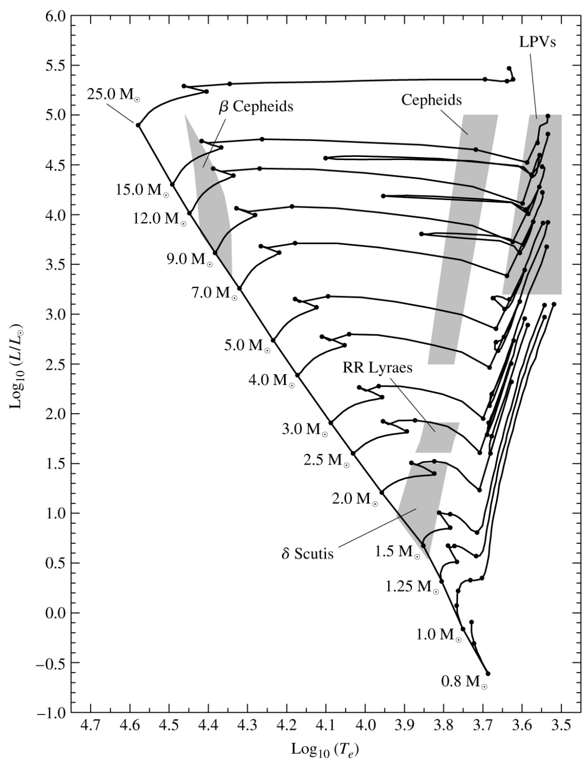

Figure 1: Pulsating star locations on the H-R diagram.

Mira or Long Period Variables

David Fabricius gets credit as the first westerner to note a variable star, that of Mira in Cetus, in 1595. The star he was observing is named “Mira,” and this class of stars is known as **“Mira variables**,” or “long period variables.” They are bright stars that have periods between 100 and 700 days.

Mira variables are AGB stars undergoing shell H and He fusion. They are variable because such shell fusion is unstable and the entire star expands and contracts. Mira variables should be less than 2 Solar masses.

Cepheids

Nearly 200 years went by before the next variable star was identified. I honestly don’t understand why it took so long! On September 10, 1784, Edward Pigott detected the variability of η Aquilae, the first known representative of the class of classical Cepheid variables. The eponymous star for classical Cepheids, δ Cephei was discovered to be variable by John Goodricke a few months later. δ Cephei, varies regularly with a period of 5 days 8 hours and 48 minutes. So-called **“Cepheids”** are incredibly important for astronomy.

Cepheids vary over periods of $\sim 1-100$ days and can be quite luminous. There are tens of thousands known.

Henrietta Swan Leavitt discovered thousands of Cepheids while working as a “computer” at Harvard. Computers were women who did work that men didn’t want to. Her task was to compare photographs to identify variable stars. She then noticed that the most luminous Cepheids have the longest periods, leading to the “period-luminosity relationship,” one of the most useful relationships in astronomy.

The period-luminosity relationship can be strengthened by observing in the infrared (“extinction” from dust is less of an issue) or by using only certain Cepheids (it turns out there are different kinds, or even by adding a color term to the fit. The most basic relation is given by the Big Orange Book in V-band as

$$M_V = -2.81 \log_{10} P_d - 1.54\,, \tag{1}$$

where the period is given in days. We can combine this relationship with the distance modulus to get a distance. Cepheids are therefore “standard candles.”

Cepheid variables are divided into two subclasses which exhibit markedly different masses, ages, and evolutionary histories: classical (type I) Cepheids and type II Cepheids.

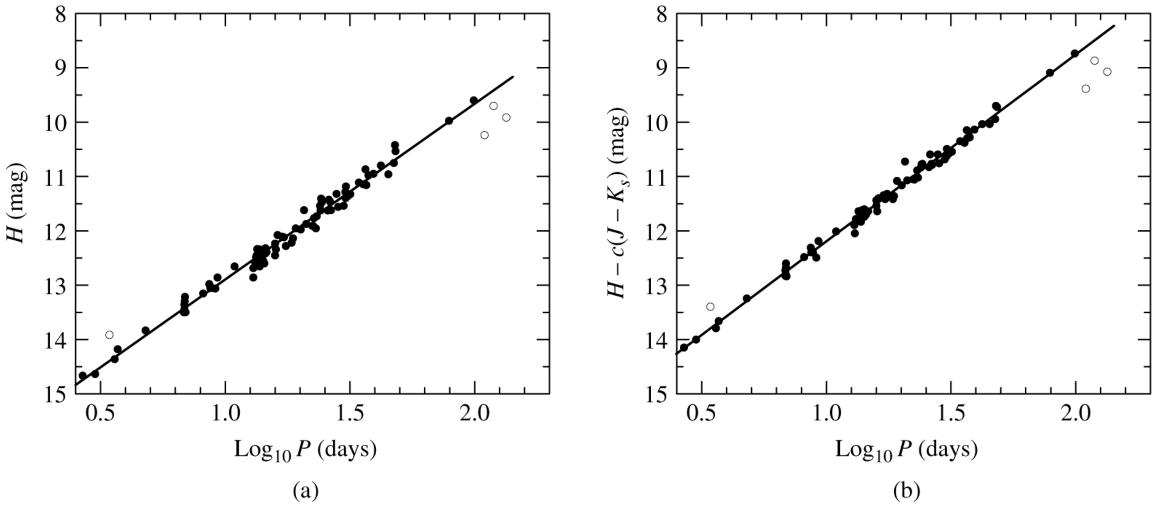

Figure 2: The Period-luminosity relation of classical Cepheids.

Type I Cepheids (Delta Cepheid variables) undergo pulsations with very regular periods on the order of days to months. Classical Cepheids are Population I variable stars which are 4-20 times more massive than the Sun, and up to 100,000 times more luminous. These Cepheids are yellow bright giants and supergiants of spectral class F6-K2 and their radii change by millions of kilometers during a pulsation cycle.

Classical Cepheids are used to determine distances to galaxies within the Local Group and beyond, and are a means by which the Hubble constant can be established. Classical Cepheids have also been used to clarify many characteristics of the Milky Way galaxy, such as the Sun's height above the galactic plane and the Galaxy's local spiral structure.

A group of classical Cepheids with small amplitudes and sinusoidal light curves are often separated out as Small Amplitude Cepheids or s-Cepheids, many of them pulsating in the first overtone.

Type II Cepheids (also termed Population II Cepheids) are population II variable stars which pulsate with periods typically between 1 and 50 days. Type II Cepheids are typically metal-poor, old ($\sim$10 Gyr), low mass objects (about half the mass of the Sun). Type II Cepheids are divided into several subgroups by period. Stars with periods between 1 and 4 days are of the BL Her subclass, 10–20 days belong to the **W Virginis** subclass, and stars with periods greater than 20 days belong to the **RV Tauri** subclass.

Anomalous Cepheids are a group of pulsating stars on the instability strip have periods of less than 2 days, similar to RR Lyrae variables but with higher luminosities. Anomalous Cepheid variables have masses higher than type II Cepheids, RR Lyrae variables, and the Sun. It is unclear whether they are young stars on a "turned-back" horizontal branch, blue stragglers formed through mass transfer in binary systems, or a mix of both.

RR Lyrae stars

RR Lyrae stars pulse in a manner similar to Cepheid variables, but the nature and histories of these stars is thought to be rather different.

RR Lyraes are old, relatively low mass, Population II stars, in common with W Virginis and BL Herculis variables, the type II Cepheids. RR Lyrae variables are much more common than Cepheids, but also much less luminous. Their period is shorter than that of Cepheids, typically less than one day, sometimes ranging down to seven hours.

Other Types of Variable Stars

Other types of variable stars are: W Virginis stars, δ Scuti stars, β Cephei stars, and ZZ Ceti stars (and many more!). These have periods ranging from 100s of seconds to days. ZZ Ceti stars are actually pulsating white dwarfs. ZZ Ceti stars pulsate due to periodic changes in the ionization and recombination of hydrogen in their outer envelopes. Cool!

The Physics of Stellar Pulsations

Just as earthquakes tell us about processes in the Earth's interior, stellar pulsations can reveal processes in stellar interiors.

How fast do stars pulsate?

We can once again arrive at a rough timescale by seeing how long it would take a pressure wave to traverse the medium (in this case, the star). Many processes are bound by this speed, and it therefore gives us a rough way of estimating timescales. Remember, we used this same line of argument when deriving the Jean's mass.

The sound speed (pressure wave speed) is

$$c_s = \sqrt{\frac{\gamma P}{\rho}}. \quad (2)$$

If we assume hydrostatic equilibrium, and further (unrealistically) assume constant density,

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\frac{G\left(4/3\pi r^3\rho\right)\rho}{r^2} = -4/3\pi G\rho^2 r\,. \tag{3}$$

We can integrate this going from the outside in, from $r = R$ to $r = r$ and from $P = 0$ to $P = P$:

$$\int_0^P dP = \int_R^r -4/3\pi G\rho^2 r dr \tag{4}$$

$$P(r) = 2/3\pi G\rho^2(R^2 - r^2)\,. \tag{5}$$

(note that this is a general expression. We didn't derive it when discussing stellar structure, because it is kind of wrong....) If we define the pulsation period as

$$\Pi \approx 2\int_0^R \frac{dr}{c_s} \approx 2\int_0^R \frac{dr}{\sqrt{2/3\gamma\pi G\rho(R^2 - r^2)}} \approx \sqrt{\frac{3\pi}{2\gamma G\rho}} \propto 1/\sqrt{\rho}\,. \tag{6}$$

So the pulsation period varies as the inverse of the density. Thus, ZZ Ceti stars, with high densities, pulsate rapidly. Note also how similar this is to the free-fall time!

Radial Modes of Pulsation

Stars pulsate radially, in a manner that is similar to that of sound waves in an organ pipe. There is a fixed knot at the star's center, or the closed end of the organ pipe, and an open end at the star's surface, or the open end of the organ pipe. For the fundamental mode, the pressure wave moves through the star radially, starting in the center. For the first overtone, there is a second node that reverses the flow. For the second overtone, there is a second node that reverses it again.

Classical Cepheids and W Viginis stars pulsate in the fundamental mode.

Eddington's Thermodynamic Heat Engine (the "κ-mechanism")

Sir Arthur Eddington (of the Eddington Limit) proposed that variable stars are actually heat engines. A heat engine converts thermal or chemical energy into mechanical energy, like the internal combustion engine of cars.

The classical Carnot engine works as follows:

- Isothermal expansion. Heat is transfered from the hot reservoir to the gas. The gas is in thermal contact with the hot temperature reservoir,

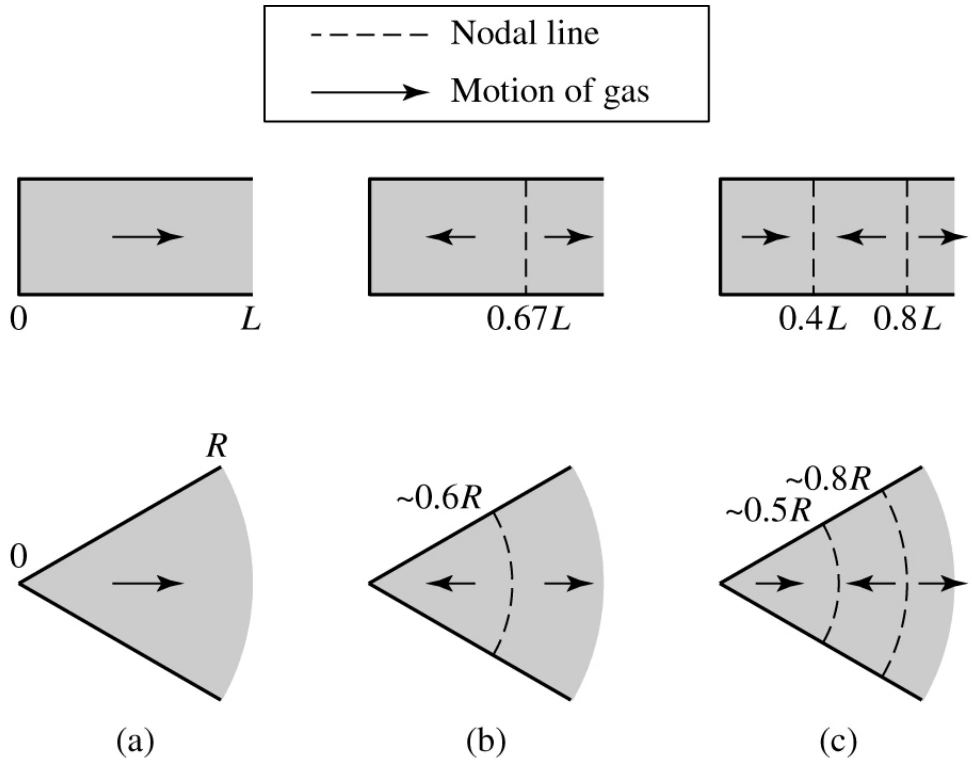

Figure 3: Radial pulsation modes for (L to R) fundamental, first overtone, and second overtones.

and is thermally isolated from the cold temperature reservoir. The gas is allowed to expand, doing work on the surroundings by pushing up the piston. The temperature of the gas does not change during the process because the heat transferred from the hot temperature reservoir to the gas is exactly used to do work on the surroundings by the gas.

- The gas in the engine is thermally insulated from both the hot and cold reservoirs, thus they neither gain nor lose heat. The gas continues to expand with reduction of its pressure, doing work on the surroundings (raising the piston), and losing an amount of internal energy equal to the work done. The loss of internal energy causes the gas to cool.

- Heat is transferred reversibly to the low temperature reservoir. The surroundings do work on the gas, pushing the piston down.

- Once again the gas in the engine is thermally insulated from the hot and cold reservoirs. During this step, the surroundings do work on the gas, pushing the piston down further (Stage Four figure, right), increasing its internal energy, compressing it, and causing its temperature to rise back to the original temperature.

If gas expands or contracts, it does PdV work. If over a cycle $\int PdV > 0$, the

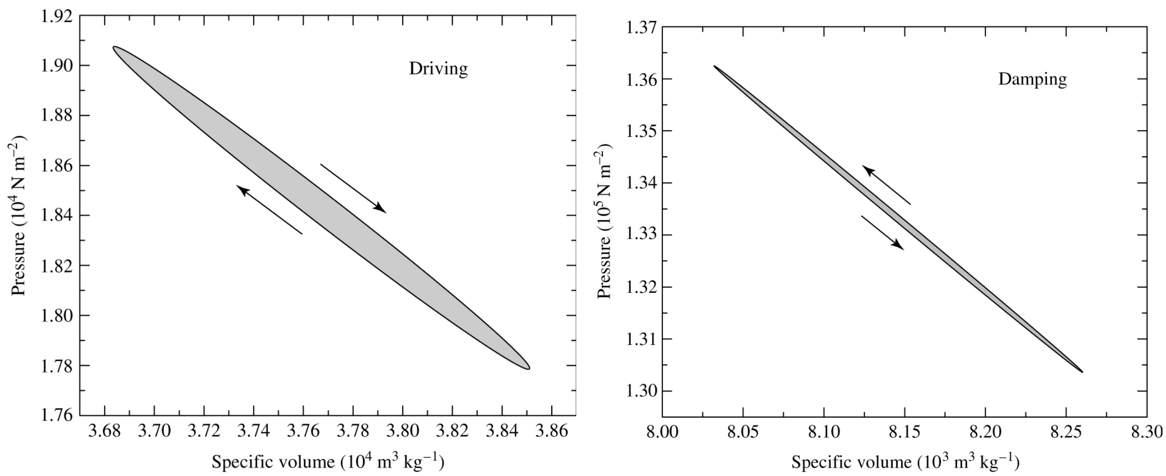

Figure 4: Eddington's Heat Engine. (left) This cycle shows a net driving engine moving in the clockwise direction. (right) This is a net damping engine moving in the counterclockwise direction.

net work is positive and the oscillations can grow in amplitude. If the net work is negative, the oscillations are damped.

The net work done by each layer of the star during one cycle is the difference between the heat flowing into the gas and the heat leaving the gas. For driving, the heat must enter during the high-temperature part of the cycle and leave during the low-temperature part. The driving layers of a pulsating star must absorb heat around the time of their maximum compression. The maximum pressure will occur after maximum compression.

Eddington's Valve

Eddington also came up with an explanation for how the heat engine may proceed. It turns out that oscillations are too small in star's cores to explain the wide range of luminosities during pulsation.

If a star becomes more opaque upon compression, energy has more trouble escaping (this is the κ in "κ-mechanism"). This energy is trapped within the star, pushing the surface layers outward. The expanding layer becomes more transparent, the heat escapes, the layer falls back down, and the cycle begins again.

This, however, is counter-intuitive. For most stellar material, $\kappa \propto \rho T^{-3.5}$, so if we assume ideal gas, $\kappa \propto \rho^{-2.5}$. We need the opacity to increase as the density increases for the heat engine to be operational. This requires more fine-tuning, which is why most stars don't pulsate.

κ and γ Mechanisms

The fine-tuning is actually just partial ionization, which allows stars to absorb energy to flow into a compressed region (the γ mechanism) and for the compressed gas to absorb this heat (the "κ" mechanism). The locations of these partial ionization zones differ depending on the size of the star.

Here, the Greek letter kappa is used to indicate the radiative opacity at any particular depth of the stellar atmosphere. In a normal star, an increase in compression of the atmosphere causes an increase in temperature and density; this produces a decrease in the opacity of the atmosphere, allowing energy to escape more rapidly. The result is an equilibrium condition where temperature and pressure are maintained in a balance.

However, in cases where the opacity increases with temperature, the atmosphere becomes unstable against pulsations. If a layer of a stellar atmosphere moves inward, it becomes denser and more opaque, causing heat flow to be checked. In return, this heat increase causes a build-up of pressure that pushes the layer back out again. The result is a cyclic process as the layer repeatedly moves inward and then is forced back out again.

Nonradial pulsation

So far we have only discussed radial pulsations where stars will pulsate in the same direction over their entire surfaces. These nonradial pulsations can be described by, you guessed it, spherical harmonics! Modes with $m = 0$ are static in time, whereas those with $m > 0$ move across the stellar surface.

Actual stars have very high m and ℓ values for their spherical harmonics, which is more akin to a bell ringing.

For radial pulsation, we can think of sound waves originating in the stellar interior. For nonradial pulsations, the sound waves can travel across the surface of the star. Since pressure is the restoring force, these are called p-modes. The acoustic frequency of a p-mode is the time it takes to travel around from one nodal line to the next:

$$\lambda = \frac{2\pi r}{\sqrt{\ell(\ell+1)}} \tag{7}$$

Stars also have g-modes, which are internal gravity waves. This is more of a sloshing effect rather than an ordered traveling across the stellar surface. The **Brunt-Vasala frequency** of **buoyancy frequency** gives us the characteristic frequency of g-mode waves:

$$N = \sqrt{\left(\frac{1}{\gamma P}\frac{dP}{dr} - \frac{1}{\rho}\frac{p\rho}{dr}\right)g} \tag{8}$$

Helioseismology

We can study the Sun's nonradial motions. A typical oscillation magnitude is only 0.1 m/s and $\delta L/L \simeq 10^{-6}$. The Big Orange Book says that there are ten million modes on the Sun!

Adding in rotation

As we had before, the Sun rotates differentially so that the rotation rate (in ω) is faster at the equator than at the poles. Additionally, the layers don't rotate homogeneously! The Big Orange Book goes into detail, but I don't see much value in considering all the complications.