Binaries
C+O Chapter 18, with info from Chapters 2 & 7

Kepler's Laws

You probably covered Kepler's Laws in Classical, so we won't do a full derivation here. Instead, let's review the three laws and briefly describe their importance. This information is in C+O, Chapter 2.

Kepler's first law states that planets orbit the Sun in an ellipse. It is often stated that the Sun is at one focus of the ellipse, which is more or less correct for the Solar system. In fact, the planet orbits around the center of mass of the Solar system as a whole. (The Sun also orbits around the center of mass.) For the Solar system, this happens to be very close to the center of the Sun.

Kepler's second law states that a planet will sweep out equal areas of its elliptical orbit in equal amounts of time. Essentially, in an elliptical orbit, at closest approach to the center of mass, the orbiting bodies speed up.

We can think of Kepler's second as an application of the Virial Theorem: $\langle 2K \rangle + \langle U \rangle = 0$. Assume two bodies and that the massive body is so much larger than the other body that we can neglect its motion. We know $U = -GMm/r$ at any given time, and $K = 1/2mv^2$, so

$$\frac{GMm}{r} = mv^2 \,. \tag{1}$$

and therefore

$$v = \left(\frac{GM}{r}\right)^{1/2} \tag{2}$$

Thus, as r goes up, v goes down, and vice-versa.

We can do a little better than this if we allow the more massive body to also move. Let's use conservation of energy. In the gravitational two-body problem, the "specific orbital energy" (or "vis-viva energy"; the energy divided by the reduced mass) of two orbiting bodies is the constant sum of their mutual potential energy and their total kinetic energy. According to the Vis-viva equation, the specific energy does not vary with time:

$$\varepsilon = \varepsilon_k + \varepsilon_p = \frac{v_a^2}{2} - \frac{GM}{r_a} = \frac{v_p^2}{2} - \frac{GM}{r_p}, \tag{3}$$

where v is the relative orbital speed, r is the orbital distance between the bodies, and the subscripts a and p refer to apiosis (closest approach) and periosis (furthest). Conserving angular momentum, $r_a v_a = r_p v_p$. We can then use an equation for ellipses $2a = r_a + r_p$ and solve for the specific energy to find (after a few lines of algebra):

$$\epsilon = -\frac{GM}{2a} \tag{4}$$

Plugging back in,

$$-\frac{GM}{2a} = \frac{v^2}{2} - \frac{GM}{r}\,, \tag{5}$$

where r is the distance between the two bodies, or

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)\,. \tag{6}$$

Note that the total energy in an orbit is negative! $E = K + U$ but in a gravitationally bound system the Virial theorem says $2K = -U$ or $K = -1/2U$, so $E = 1/2U = -GMm/2r$. Unbound systems have zero or even positive energy.

Kepler's third law is the big one. Although his first and second laws appeared together, it took Kepler ten more years to publish his third law. The general form of Kepler's third is:

$$P^2 = \frac{4\pi^2}{G(M+m)}a^3\,, \tag{7}$$

with P the period and a the semimajor axis. If P is measured in years, and a in AU, we simply have $P^2 = a^3$. Note that $M + m \simeq M$ for the Solar system, but not in general for stars.

Kepler's third says that bodies orbiting far from the center of mass take longer to complete their orbits. For example, between when Pluto was discovered and when it was reclassified as a dwarf planet, it hadn't yet completed a full orbit!

Types of binaries (Ch. 7)

At least half of all stars are binaries (my grad school prof used to say "three out of every two stars is a binary" but it was never clear if he was being funny or was confused). The fraction of binaries seems to increase with increasing mass. Binary stars can have different evolutionary paths from single stars, especially if they are close together and can share mass.

Binaries are incredibly important because we can use them to determine the properties of the stars themselves. It's difficult to determine the properties of single stars, but with two stars orbiting, we can use their orbital period and separations to measure masses (and luminosities).

Like most things in astronomy, we can classify binaries by the observed characteristics.

- **Optical doubles** are not true binaries - they just appear close to each other in the sky.

- **Visual binaries** are binaries for which we can see both stars. We can see the motion of both stars.

- **Astrometric binaries** are binaries for which we cannot see both members, only the brighter one. Instead, we can see the brighter one shifting in position.

- **Eclipsing binaries** are binaries for which we see the light of each star regularly dimming. Sometimes we can only see one of the two stars. The shape of the "light curves" hold tons of information about the stars themselves. The geometry must be correct so that one star can pass in front of the other.

- **Spectrum binaries** are binaries for which we can separate the stars based on their spectra alone. We see spectral lines shifting back and forth. The geometry must be correct.

- **Spectroscopic binaries** are those for which we can only see one star's spectrum Doppler shifted.

Masses of Visual Binaries

The separation of binaries from their mutual center of mass is related to the masses themselves. We know that the more massive star does not stray much from the center of mass. We can write

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}\,, \tag{8}$$

where a_1 and a_2 are the semimajor axes of the two stars.

We, however, observe *angular* distances instead of real distances. Luckily, we can use those instead. Using the angle $\alpha = a/d$, where d is the distance to the system, or $a = d\alpha$, and since the distance to each star is the same,

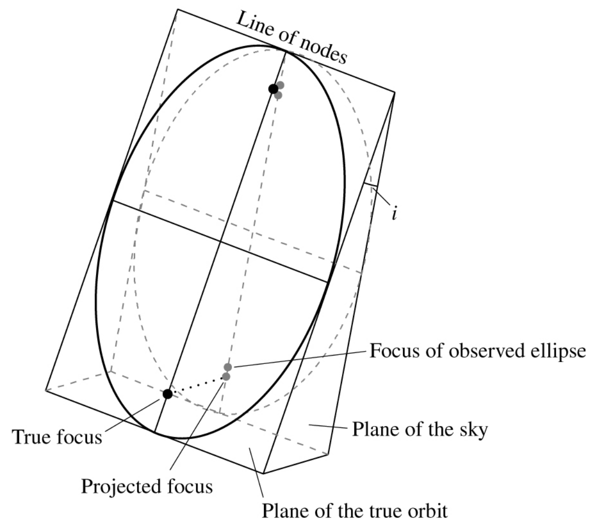
$$\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}\,. \tag{9}$$

Therefore, we can still find the summed masses even if the distance to the system is unknown.

We know from Kepler's third law that the orbital period P is related to the semimajor axis a. Both stars share the same period, but different a. What a to use? I'll leave it as an exercise to show that $a = a_1 + a_2$. Therefore, we can in principle use the measured semimajor axes and period to get the individual masses.

One issue is that the orbits are inclined relative to the plane of the sky. We define this inclination i as $i = 0$ (face-on) when the orbit is in the plane of the sky and $i = 90\,^\circ$ (edge-on) when it is perpendicular. We actually measure angle $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$. From the definition of the inclination, $\tilde{\alpha}_1 = \alpha_1 \cos i$ and $\tilde{\alpha}_2 = \alpha_2 \cos i$. Therefore, the inclination again doesn't matter for the mass ratio:

$$\frac{m_1}{m_2} = \frac{\tilde{\alpha}_2}{\tilde{\alpha}_1} \tag{10}$$



From the above relations, we can also write that $\alpha = a/d$ and $\tilde{\alpha} = \tilde{\alpha}_1 + \tilde{\alpha}_2$.

This is all well and good, but it does make our lives a bit more complicated. We can rearrange Kepler's 3rd to get

$$m_1 + m_2 = \frac{4\pi^2}{G} \frac{a^3}{P^2} \tag{11}$$

We know that $a = \alpha d$ and $\alpha = \tilde{\alpha}/\cos i$. Therefore,

$$m_1 + m_2 = \frac{4\pi^2}{G} \left(\frac{d}{\cos i} \right)^3 \frac{\tilde{\alpha}^3}{P^2} \tag{12}$$

This form of Kepler's third is appropriate for binaries, but it depends on determination of d, $\tilde{\alpha}$ and P. We can help ourselves out by also measuring radial velocities.

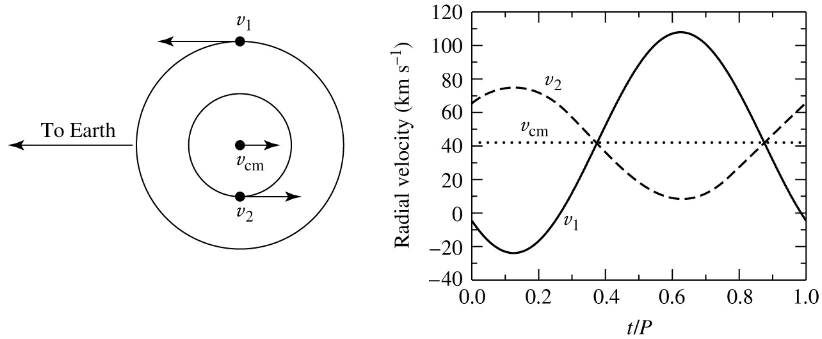
Spectroscopic Binaries

If we can measure the spectral lines from binaries, there is a wealth of additional information we can easily obtain. Assume the stars have velocities v_1 and v_2. Again, the inclination comes into play, except this time if $i = 0$ we have no radial velocity. Therefore, $v_\mathrm{r} = v \sin i$.

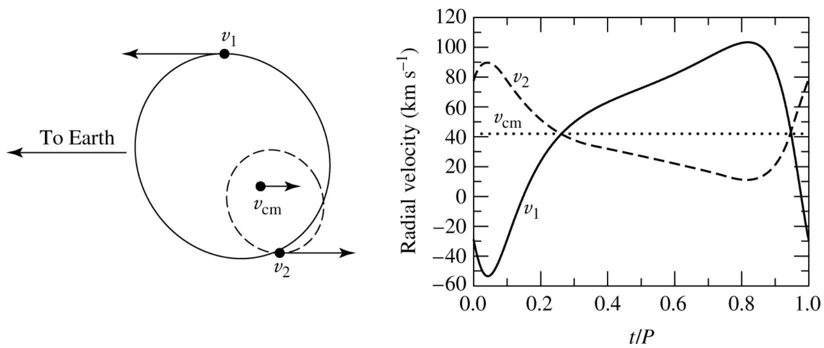
Furthermore, if the orbits are eccentric, the velocity curves get skewed. We won't deal much with this scenario.

Let's assume very small eccentricity. Then the stellar speeds are unchanging and we can write $v_1 = 2\pi a_1/P$ and $v_2 = 2\pi a_2/P$. Therefore,

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} \tag{13}$$



(a) (b)



(a) (b)

Just as before, our trig factors of inclination cancel, so

$$\frac{m_1}{m_2} = \frac{v_{2,\mathrm{r}}}{v_{1,\mathrm{r}}} \tag{14}$$

Just like for visual binaries, if we want to know the sum of the masses we need to know the angle of inclination.

$$a = a_1 + a_2 = \frac{P}{2\pi}(v_1 + v_2)\,, \tag{15}$$

so

$$m_1 + m_2 = \frac{P}{2\pi G}(v_1 + v_2)^3\,. \tag{16}$$

Or, in terms of observables,

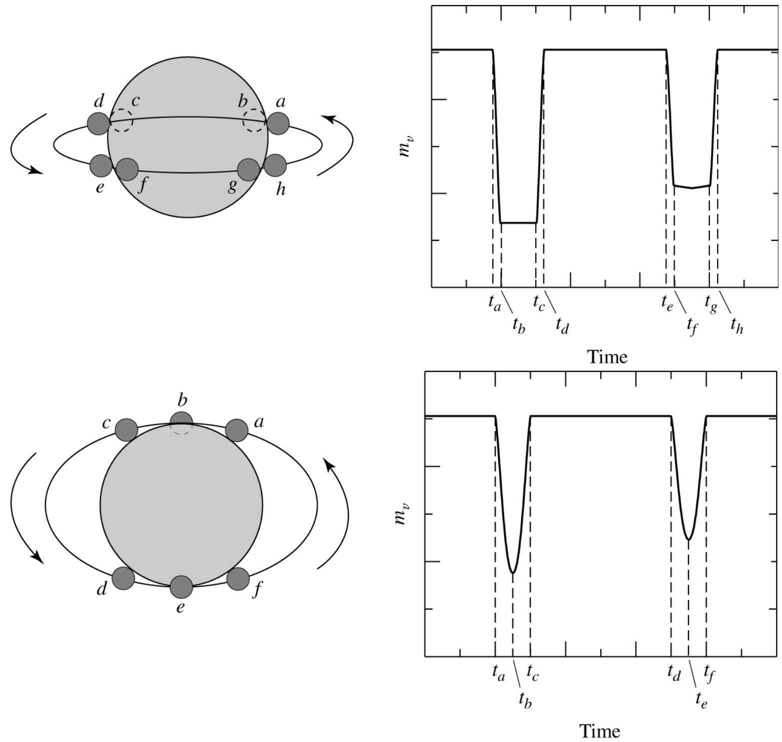
$$m_1 + m_2 = \frac{P}{2\pi G}\frac{(v_{1,\mathrm{r}} + v_{2,\mathrm{r}})^3}{\sin^3 i}\,. \tag{17}$$

This relies on the determination of both radial velocities, which is often not possible. If only one is measured, we can use the relationship between the masses and velocities:

$$m_1 + m_2 = \frac{P}{2\pi G}\frac{v_{1,\mathrm{r}}^3}{\sin^3 i}\left(1 + \frac{m_1}{m_2}\right)^3\,. \tag{18}$$

or

$$\frac{m_2^3}{(m_1 + m_2)^2}\sin^3 i = \frac{P}{2\pi G}v_{1,\mathrm{r}}^3\,. \tag{19}$$



The RHS is observable quantities. This is known as the "mass function." We cannot know about mass ratios with only one star. Also, the inclination messes us up. The book states that an average value of $\sin^3 i \simeq 2/3$. If m_1 or $\sin i$ is unknown, the mass function sets a lower limit for m_2 since the LHS is always less than m_2.

Eclipsing Binaries

As mentioned above, the inclination angle is often an illusive parameter to determine. For eclipsing binaries, however, we know that the inclination must be close to $90\,^\circ$, and so these complications are minimized. We can use the light curves of eclipsing binaries to refine i further, since the shape of the curves is dependent on i. If the secondary star passes entirely in front of the primary, the transiting portion of the light curve will be flat bottomed. This implies and inclination very close to $90\,^\circ$. If the light curve is flat bottomed, the inclination is slightly less. Note that small uncertainties in i don't change $\sin^3 i$ very much.

We also learn of the radius of the eclipsing pair. A larger secondary will create a larger eclipse of course. For circular orbits,

$$r_s = \frac{v}{2}(t_b - t_a)\,, \tag{20}$$

where r_s is the radius of the secondary, v is the relative velocity between the primary and secondary, t_b is the time of minimum light, and t_a is the time of first contact. We can also get the radius of the primary star

$$r_p = \frac{v}{2}(t_c - t_a) = r_s + \frac{v}{2}(t_c - t_b)\,, \tag{21}$$

where t_c is when the secondary begins to end its transit. To make use of these equations, we need to determine the velocity from spectra.

Transits also allow us to compute relative temperatures. For blackbodies, we can easily relate the intensity to the surface area. Since stars are basically blackbodies, the same prescription applies. During an eclipse, the secondary contributes light, but less if it is cooler.

If we assume that the stellar discs are at a constant surface brightness, when we can see both stars, we get intensity

$$B_0 = k(\pi r_p^2 F_p + \pi r_s^2 F_s)\,, \tag{22}$$

where r is the radii and F is the flux, $F = \sigma T^4$. k is a constant that depends on the distance to the system and the amount of intervening material that absorbs and scatters light. In the deeper of primary minimum in the light curve,

$$B_p = k\pi r_p^2 F_p \tag{23}$$

and in the shallower secondary minimum,

$$B_s = k(\pi r_p^2 - \pi r_s^2) f_p + k\pi r_s^2 F_s\,. \tag{24}$$

Since we don't usually know k, we can instead write

$$\frac{B_0 - B_p}{B_0 - B_s} = \frac{F_s}{F_p} = \left(\frac{T_s}{T_p}\right)\,. \tag{25}$$

Therefore, we can easily estimate the relative temperatures of the two stars from the light curves. We can get the radii of the stars from the light curves as well. The shape of the curves gets us the inclination angles, and if we also have velocity information we can get the sum of the masses. Binaries thus give us windows into stellar systems that we don't otherwise have.

Exoplanets

The same treatment of binaries applies equally well to exoplanets. Our book is too old to do a full treatment of exoplanets, but I think it's important to spend some time reviewing the state of the field. I'm just going to steal this information from Wikipedia, because I'm lazy.

As for binaries, there are two main tools we can use to detect exoplanets: photometry and spectroscopy. For photometric measurements, we can sometimes see shifts in the position of the stars, sometimes directly image the planets, and sometimes detect transits. For spectroscopy, we can measure the radial velocity changes of the star. These methods provide complimentary information.

The first confirmation of detection occurred in 1992. This was followed by the confirmation of a different planet, originally detected in 1988. As of 1 November 2019, there are 4,126 confirmed exoplanets in 3,067 systems, with 671 systems having more than one planet.

There is at least one planet on average per star. About 1 in 5 Sun-like stars have an "Earth-sized" planet in the habitable zone. Assuming there are 200 billion stars in the Milky Way, it can be hypothesized that there are 11 billion potentially habitable Earth-sized planets in the Milky Way, rising to 40 billion if planets orbiting the numerous red dwarfs are included.

Initially, most known exoplanets were massive planets that orbited very close to their parent stars. Astronomers were surprised by these "hot Jupiters", because theories of planetary formation had indicated that giant planets should only form at large distances from stars. But eventually more planets of other sorts were found, and it is now clear that hot Jupiters make up the minority of exoplanets. In 1999, Upsilon Andromedae became the first main-sequence star known to have multiple planets. Kepler-16 contains the first discovered planet that orbits around a binary main-sequence star system.

Most known exoplanets orbit stars roughly similar to the Sun, i.e. main-sequence stars of spectral categories F, G, or K. Lower-mass stars (red dwarfs, of spectral category M) are less likely to have planets massive enough to be detected by the radial-velocity method. Despite this, several tens of planets around red dwarfs have been discovered by the Kepler spacecraft, which uses the transit method to detect smaller planets.

Using data from Kepler, a correlation has been found between the metallicity of a star and the probability that the star host planets. Stars with higher metallicity are more likely to have planets, especially giant planets, than stars with lower metallicity.

Some planets orbit one member of a binary star system, and several circumbinary planets have been discovered which orbit around both members of binary star. A few planets in triple star systems are known and one in the quadruple system Kepler-64.

As more planets are discovered, the field of exoplanetology continues to grow into a deeper study of extrasolar worlds, and will ultimately tackle the prospect of life on planets beyond the Solar System. At cosmic distances, life can only be detected if it is developed at a planetary scale and strongly modified the planetary environment, in such a way that the modifications cannot be explained by classical physico-chemical processes (out of equilibrium processes). For example, molecular oxygen in the atmosphere of Earth is a result of photosynthesis by living plants and many kinds of microorganisms, so it can be used as an indication of life on exoplanets, although small amounts of oxygen could also be produced by non-biological means. Furthermore, a potentially habitable planet must orbit a stable star at a distance within which planetary-mass objects with sufficient atmospheric pressure can support liquid water at their surfaces.

https://en.wikipedia.org/wiki/Exoplanet

Chapter 18

Lagrangian points

In celestial mechanics, the Lagrangian points are the points near two large bodies in orbit where a smaller object will maintain its position relative to the large orbiting bodies. At other locations, a small object would go into its own orbit around one of the large bodies, but at the Lagrangian points the gravitational forces of the two large bodies, the centripetal force of orbital motion, and (for certain points) the Coriolis acceleration all match up in a way that cause the small object to maintain a stable or nearly stable position relative to the large bodies.

There are five such points, labeled L1 to L5, all in the orbital plane of the two large bodies, for each given combination of two orbital bodies. For instance, there are five Lagrangian points L1 to L5 for the Sun-Earth system, and in a similar way there are five different Lagrangian points for the Earth-Moon system. L1, L2, and L3 are on the line through the centers of the two large bodies. L4 and L5 each form an equilateral triangle with the centers of the large bodies. L4 and L5 are stable, which implies that objects can orbit around them in a rotating coordinate system tied to the two large bodies.

In contrast to L4 and L5, where stable equilibrium exists, the points L1, L2, and L3 are positions of unstable equilibrium. Any object orbiting at L1, L2, or L3 will tend to fall out of orbit; it is therefore rare to find natural objects there, and spacecraft inhabiting these areas must employ station keeping in order to maintain their position.

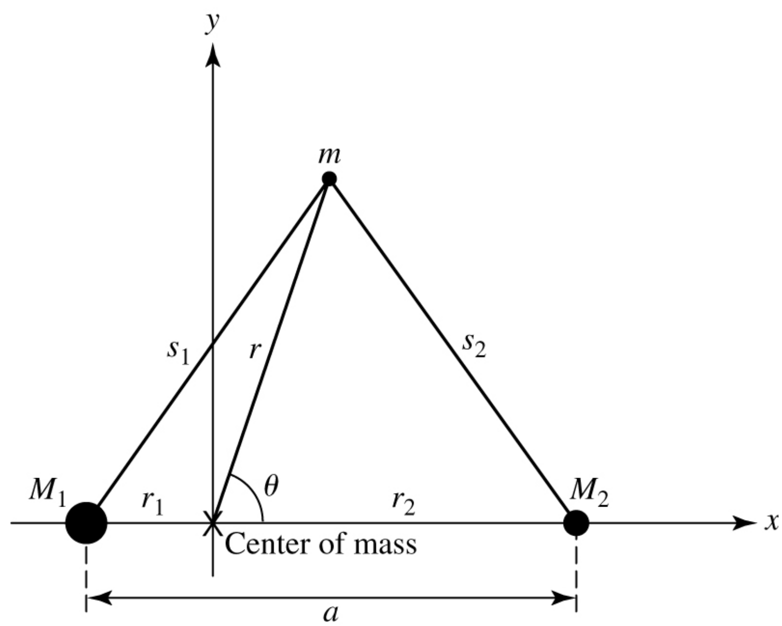
Several planets have trojan satellites near their L4 and L5 points with respect to the Sun. Jupiter has more than a million of these Trojans. Artificial satellites have been placed at L1 and L2 with respect to the Sun and Earth, and with respect to the Earth and the Moon. The Lagrangian points have been proposed for uses in space exploration.

The Lagrangian points are maxima and minima of the effective potential, which is the gravitational potential ($U_g = -G\frac{M_1 M_2}{r}$) and the centrifugal force. We can define a fictitous centrifugal potential energy

$$U_f - U_i = \Delta U_c = -\int_{r_i}^{r_f} F_c \cdot dr = -\int_{r_i}^{r_f} m\omega^2 r dr = -\frac{1}{2}m\omega^2(r_f^2 - r_i^2)\,. \tag{26}$$

We can arbitrarily define the zero-point of centrifugal potential at $r = 0$ to get

$$U_c = -\frac{1}{2}m\omega^2 r^2 \tag{27}$$



If we define $r_1 + r_2 = a$ and $M_1 r_1 = M_2 r_2$, we can write

$$U = -G\left(\frac{M_1 M_2}{s_1} + \frac{M_1 M_2}{s_2}\right) - \frac{1}{2}m\omega^2 r^2 \tag{28}$$

The gravitational potential maxima and minima are when

$$F_x = -\frac{dU}{dx} = 0 \tag{29}$$

These are three of the Lagrangian points.

Lagrange points

L1 point
The L1 point lies on the line defined by the two large masses M1 and M2, and between them. It is the most intuitively understood of the Lagrangian points: the one where the gravitational attraction of M2 partially cancels M1's gravitational attraction. An object that orbits the Sun more closely than Earth would normally have a shorter orbital period than Earth, but that ignores the effect of Earth's own gravitational pull. If the object is directly between Earth and the Sun, then Earth's gravity counteracts some of the Sun's pull on the object, and therefore increases the orbital period of the object.

The location of L1 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(a-s_2)^2} = \frac{M_2}{s_2^2} + \frac{M_1}{a^2} - \frac{s_2(M_1+M_2)}{a^3} \tag{30}$$

Solving this for a involves solving a quintic function, but if the mass of the smaller object is much smaller than the mass of the larger object then L1 and L2 are at approximately equal distances from the smaller object given by:

$$s_2 \approx a\left(\frac{M_2}{3M_1}\right)^{1/3} \tag{31}$$

L2 point
The L2 point lies on the line through the two large masses, beyond the smaller of the two. Here, the gravitational forces of the two large masses balance the centrifugal effect on a body at L2. On the opposite side of Earth from the Sun, the orbital period of an object would normally be greater than that of Earth. The extra pull of Earth's gravity decreases the orbital period of the object, and at the L2 point that orbital period becomes equal to Earth's.

The location of L2 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(a+s_2)^2} + \frac{M_2}{s_2^2} = \frac{M_1}{a^2} + \frac{s_2\,(M_1+M_2)}{a^3} \tag{32}$$

with parameters defined as for the L1 case. Again, if the mass of the smaller object is much smaller than the mass of the larger object then L2 is given by:

$$s_2 \approx a\left(\frac{M_2}{3M_1}\right)^{1/3} \tag{33}$$

L3 point
The L3 point lies on the line defined by the two large masses, beyond the larger of the two. Within the Sun-Earth system, the L3 point exists on the opposite side of the Sun, a little outside Earth's orbit and slightly further from the Sun than Earth is. This placement occurs because the Sun is also affected by Earth's gravity and so orbits around the two bodies' barycenter, which is well inside the body of the Sun. An object at Earth's distance from the Sun would have an orbital period of one year if only the Sun's gravity is considered. But an object on the opposite side of the Sun from Earth and directly in line with both "feels" Earth's gravity adding slightly to the Sun's and therefore must orbit a little further from the Sun in order to have the same 1-year period.

The location of L3 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(a-s_2)^2}+\frac{M_2}{(2a-s_2)^2}=\left(\frac{M_2}{M_1+M_2}a+a-s_2\right)\frac{M_1+M_2}{a^3} \tag{34}$$

If the mass of the smaller object is much smaller than the mass of the larger object then:

$$a \approx s_2 + a\left(\frac{5M_2}{12M_1}\right) \tag{35}$$

L4 and L5 points
The L4 and L5 points lie at the third corners of the two equilateral triangles in the plane of orbit whose common base is the line between the centers of the two masses, such that the point lies behind (L5) or ahead (L4) of the smaller mass with regard to its orbit around the larger mass.

The triangular points (L4 and L5) are stable equilibria, provided that the ratio of M1 to M2 is greater than 24.96. This is the case for the Sun-Earth system, the Sun-Jupiter system, and, by a smaller margin, the Earth-Moon system.

The reason these points are in balance is that, at L4 and L5, the distances to the two masses are equal. Accordingly, the gravitational forces from the two massive bodies are in the same ratio as the masses of the two bodies, and so the resultant force acts through the barycenter of the system; additionally, the geometry of the triangle ensures that the resultant acceleration is to the distance from the barycenter in the same ratio as for the two massive bodies. The barycenter being both the center of mass and center of rotation of the three-body system, this resultant force is exactly that required to keep the smaller body at the Lagrange point in orbital equilibrium with the other two larger bodies of system. (Indeed, the third body need not have negligible mass.) The general triangular configuration was discovered by Lagrange in work on the three-body problem.

It is common to find objects at or orbiting the L4 and L5 points of natural orbital systems. These are commonly called ”trojans”. In the 20th century, asteroids discovered orbiting at the Sun-Jupiter L4 and L5 points were named after characters from Homer's Iliad. Asteroids at the L4 point, which leads Jupiter, are referred to as the ”Greek camp”, whereas those at the L5 point are referred to as the ”Trojan camp”. The Earth has one asteriod at L4 and clumps of interplanetary dust. Several asteroids also orbit near the Sun-Jupiter L3 point, called the Hilda family. The Saturnian moon Tethys has two smaller moons in its L4 and L5 points, Telesto and Calypso. The Saturnian moon Dione also has two Lagrangian co-orbitals, Helene at its L4 point and Polydeuces at L5.

Mars has four known co-orbital asteroids (5261 Eureka, 1999 UJ7, 1998 VF31, and 2007 NS2), all at its Lagrangian points.

L4 and L5 These are harder and won't be derived!

Sun-Earth L2 is a good spot for space-based observatories. Solar and Heliospheric Observatory (SOHO) is stationed in a halo orbit at L1, and the Advanced Composition Explorer

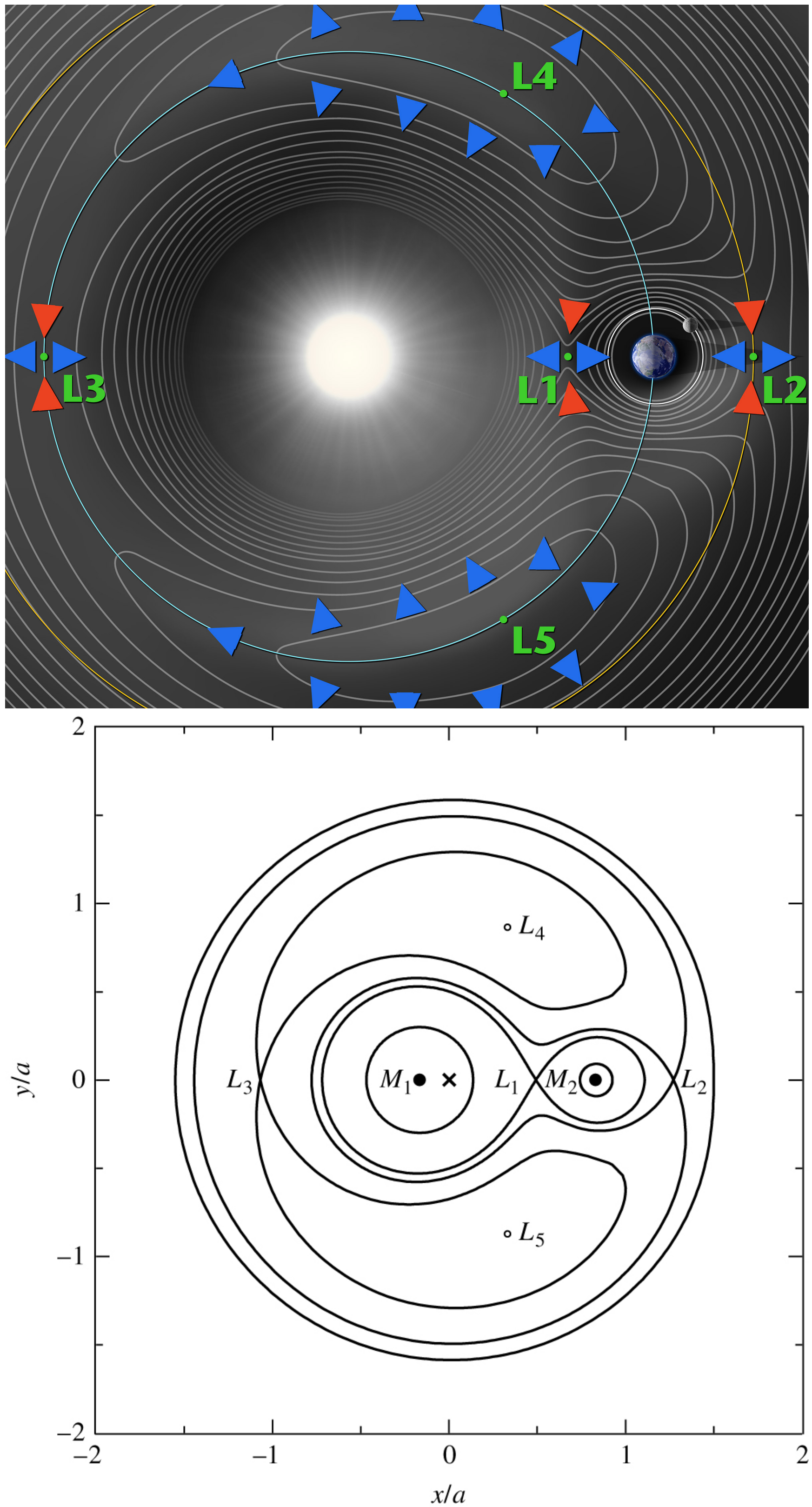


Figure 1: A contour plot of the effective potential due to gravity and the centrifugal force of a two-body system in a rotating frame of reference. The arrows indicate the gradients of the potential around the five Lagrange pointsdownhill toward them (red) or away from them (blue). Counterintuitively, the L4 and L5 points are the high points of the potential. At the points themselves these forces are balanced.

(ACE) in a Lissajous orbit. WIND is also at L1.

LISA Pathfinder (LPF) was launched on 3 December 2015, and arrived at L1 on 22 January 2016, where, among other experiments, it tested the technology needed by (e)LISA to detect gravitational waves. LISA Pathfinder used an instrument consisting of two small gold alloy cubes.

Spacecraft at Sun-Earth L2
1 October 2001 - October 2010: Wilkinson Microwave Anisotropy Probe
July 2009 - 29 April 2013: Herschel Space Telescope
3 July 2009 - 21 October 2013: Planck Space Observatory
January 2014 - 2018: Gaia Space Observatory
2021: James Webb Space Telescope will use a halo orbit
2024: Wide Field Infrared Survey Telescope (WFIRST) will use a halo orbit

Accretion Disks

Let's think about mass transfer! What happens for 1) two equal mass main sequence stars, 2) a red giant and an equal mass main sequence star, 3) a red giant and a white dwarf?

In a close binary system, mass can transfer from the lower mass star to the higher mass star through L1. The inner equipotential surface makes a figure eight, and if this surface is entirely filled with gas being stripped off M2, then we call them "contact' binaries. If only the primary surface is filled, as call that "semidetached", and if nothing is filled we call it "detached." These figure-eight surfaces are called 'Roche Lobes," and we will discuss them further next semester.

Let's assume we have a close binary. Gravity will pull the outer layers of one star onto the other. The specifics of this mass transfer depend on how tightly bound the outer layers are, which in turn depends on the mass and evolutionary stages of the two stars. The mass transfer rate can be expressed as

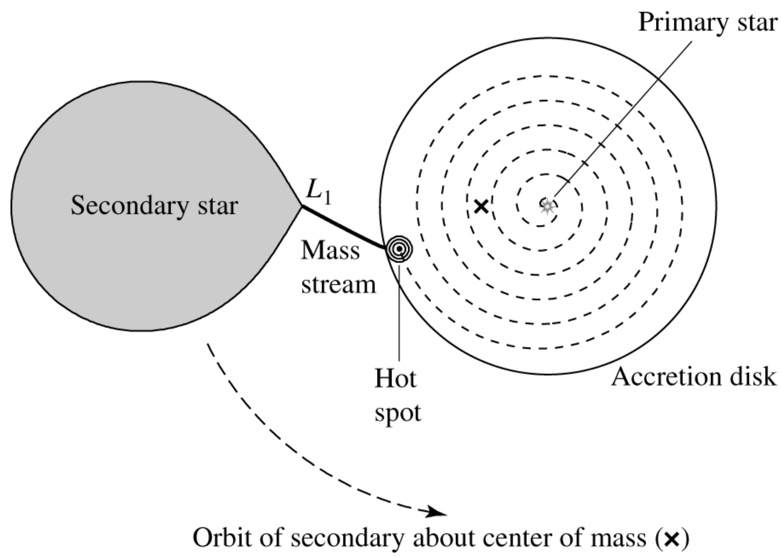
$$\dot{M} = \rho v A\,. \tag{36}$$

If we assume that the mass transfer velocity happens at the rms velocity of the gas, $v = (3kT/(\mu m_H))^{1/2}$, with a bit of geometry we find

$$\dot{M} \propto d^3 \rho T^{1/2}\,, \tag{37}$$

where d is the "overflow distance," which is inversely proportional to the distance between the stellar centers (strange way to parameterize, I know). Therefore, the mass transfer rate increases strongly as a function of stellar distance, as expected.

Material that is accreted onto the primary does so in an accretion disk, a thin disk of material that surrounds the primary. Accretion disks form because the primary is in orbit around



the CoM. The material must lose energy in order to spiral into the primary. It does so by converting orbital energy into thermal energy, heating up in the process. Therefore, we would expect that accretion disks have temperature gradients from outside in. The heating comes from friction of the gas, and possibly magnetohydrodynamics effects.

We can compute the temperature and luminosity profiles of accretion disks by assuming that they emit (optically thin) blackbody emission. If we use the Viial Theorem, $2K + U = 0$, or $K = -U/2$. Therefore, $E + K + U = 1/2U$. We can define U in the normal way,

$$E = -G\frac{M_1 m}{2r}\,, \tag{38}$$

where m is the particle mass at radius r.

Because the temperature must change throughout the disk, we want to treat annular rings of differential energy dE:

$$dE = \frac{dE}{dr}dr = \frac{d}{dr}\left(-G\frac{M_1 m}{2r}\right)dr = G\frac{M_1 \dot{M} dt}{2r^2}dr\,, \tag{39}$$

where $m = \dot{M}t$ is the mass change in the annnular ring. Since $dE = dLdt$,

$$dLdt = dE = G\frac{M_1 \dot{M} dt}{2r^2}dr \tag{40}$$

The area of the rings are $A = 2(2\pi r dr)$ and from Stephan-Boltzmann $L = A\sigma T^4$ so

$$dL = 4\pi r\sigma T^4 dr = G\frac{M_1 \dot{M}}{2r^2}dr \tag{41}$$

Therefore,

$$T = \left(\frac{GM_1\dot{M}}{8\pi\sigma R^3}\right)^{1/4}\left(\frac{R}{r}\right)^{3/4}\,, \tag{42}$$

where R is the primary's radius. Now, this is kind of a stupid way to write things because we have introduced R unnecessarily. That's because the characteristic disk temperature is often given as

$$T_{\rm disk} = \left(\frac{GM_1\dot{M}}{8\pi\sigma R^3} \right)^{1/4} \tag{43}$$

It can be shown that this is about double the maximum temperature. If we have a mass transfer rate of $1.6 \times 10^{-10}\, M_\odot {\rm yr}^{-1}$ onto a 0.85 $M_\odot$ white dwarf with $R = 0.0095 R_\odot$,

$$T_{\rm max} \simeq 1/2 T_{\rm disk} = 2.62 \times 10^4 \text{ K}. \tag{44}$$

This peks at 111 nm, in the ultraviolet. For a NS, your book finds $T_{\rm max} = 6.86 \times 10^6$ K and a peak in the X-ray.

We can integrate the luminosity

$$L_{\rm disk} = \int_{r=R}^{r=\infty} dL = G\frac{M\dot{M}}{2R} \tag{45}$$

This is the total luminosity emitted from the disk, but this only represents the half the energy of the disk, so we can write

$$L_{\rm acc} = G\frac{M\dot{M}}{R} \tag{46}$$

If we have a mass transfer rate of $1.6 \times 10^{-10}\, M_\odot {\rm yr}^{-1}$ onto a 0.85 $M_\odot$ white dwarf with $R = 0.0095 R_\odot$,

$$L_{\rm disk} = 8.55 \times 10^{25}\,{\rm W} = 8.55 \times 10^{32}\,{\rm erg/s} \tag{47}$$

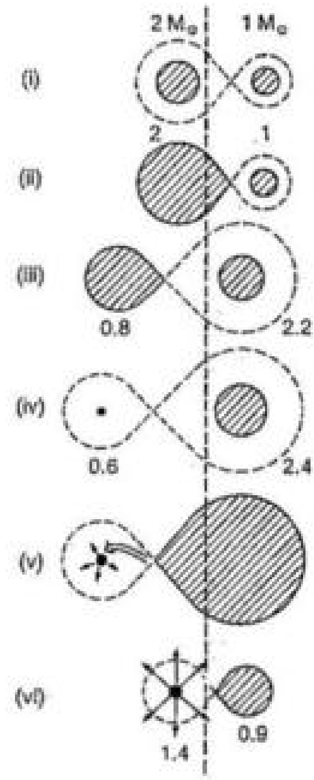
For a NS, your book finds $L_{\rm disk} = 9.29 \times 10^{29}$ W.

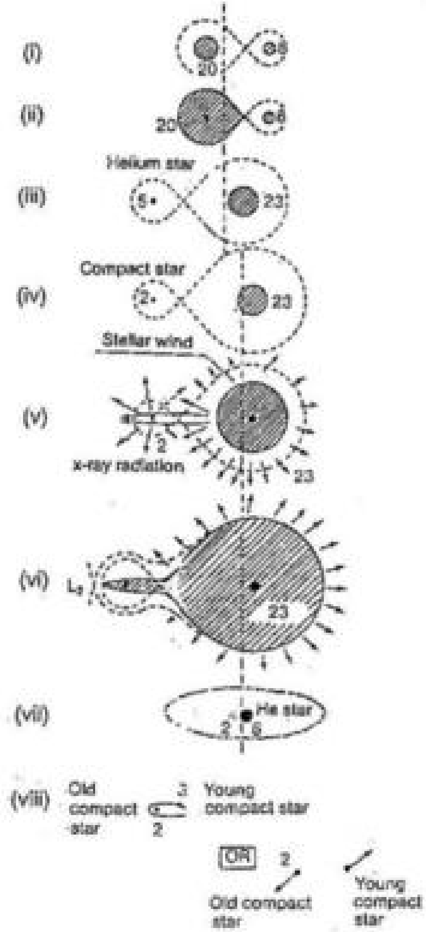
Binary Evolution

[see figure] It gets complicated! Mass transfer can affect the evolution, leading to complicated scenarios.

Novae and Supernovae

A 'nova" is just a temporary brightening of a star, and is Latin for "new." The term nova encompasses a wide variety of physical scenarios, because it is simply defined in terms of an observed change in brightness. We can separate processes that destroy the stars (supernova) from those that do not (dwarf and classical novae). All of these processes usually involve white dwarfs. Dwarf and classical novae result from "cataclysmic variables."


2 M☉
1 M☉
(i)
2
1
(ii)
(iii)
0.8
2.2
(iv)
0.6
2.4
(v)
(vi)
1.4
0.9


(i)
20
8
(ii)
20
8
Helium star
(iii)
5
23
Compact star
(iv)
2
23
Stellar wind
(v)
2
x-ray radiation
23
(vi)
L2
23
(vii)
He star
2
6
(viii)
Old compact star
3
Young compact star
2
OR
2
Old compact star
Young compact star

Dwarf Novae

In a dwarf nova, the system brightens by perhaps an order of manitude, with a full range of factors of 6 to 250. A dwarf nova is the result of brightening in the accretion disk. The brightening begins in the outer (cooler) layers of the disk, then progresses toward the primary star. The evidence for this is that we see the brightening first in the optical, then in the UV. Since the brightening begins in the outer parts of the disk, dwarf nova must be the result of a change in the mass accretion rate.

The luminosity is proportional to $\dot{M}$, so this implies that $\dot{M}$ must be increasing by an order of magnitude. Astronomers have worked out that during normal accretion $\dot{M} \approx 10^{-11} - 10^{-10}\ M_\odot$/yr. This increases to $\dot{M} \approx 10^{-8} - 10^{-9}\ M_\odot$/yr. Why this occurs is a bit difficult to understand, but your book as some possible explanations.

Classical Novae

A classical nova increases the brightness of the system by 7-20 magnitudes, or $3-8$ orders of magnitude. This brightening takes a few days, then declines in brightness slowly over several months. When the bolometric luminosity is computed, however, it is relatively constant with time for several months following the outburst.

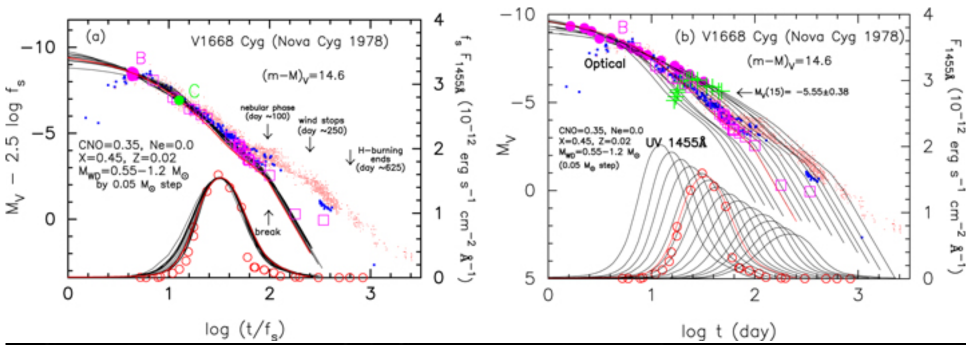
Let's assume we have a RG and a WD. What type of material is accreted onto the WD? It is the outer layers of the RG. If fully convective, this is rich in CNO. If not, less rich, but is still mostly hydroen either way.

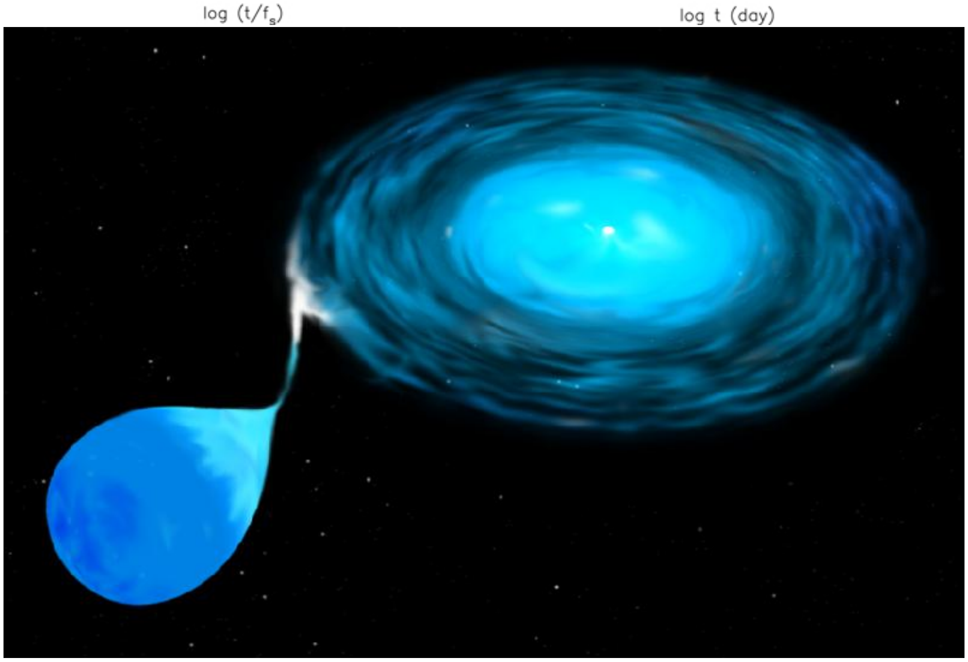
What happens when you dump a bunch of hydrogen onto a 10^6 K WD? Once there is enough of it, it will ignite in fusion reactions of primarily the CNO cycle. This limit is $10^{-4} - 10^{-5}\ M_\odot$.

Once fusion has begun, a star would normally expand. But degenerate matter does not, since the pressure is independent of the temperature. Instead, the fusion is uninhibited, and will proceed up to the Eddington luminosity, at which point the outer layers will be flung off into space. Only about 10% of the accreted H is ejected, the rest continuing to fuse until it too runs out or is expelled in subsequent ejection events. Then , the cycle can begin again.It takes $10^4 - 10^5$ years to again build up enough H.

Type Ia Supernovae

Let's review our information about Type Ia SN that we learned before. Type IA SN are caused when a WD surpasses the Chandrasekhar limit of 1.44 $M_\odot$. At this point, electron degeneracy pressure is not great enough to support the WD and it collapses, igniting runaway


(a)
V1668 Cyg (Nova Cyg 1978)
B
C
$(m-M)_V=14.6$
nebular phase (day ~100)
wind stops (day ~250)
H-burning ends (day ~625)
CNO=0.35, Ne=0.0
X=0.45, Z=0.02
$M_{WD}=0.55-1.2\ M_\odot$ by 0.05 $M_\odot$ step
break
$M_V - 2.5 \log f_s$
$\log (t/f_s)$
$f_s\ F_{1455Å}\ (10^{-12}$ erg s^{-1} cm^{-2} Å$^{-1})$
(b) V1668 Cyg (Nova Cyg 1978)
B
Optical
$(m-M)_V=14.6$
$M_V(15)= -5.55\pm0.38$
CNO=0.35, Ne=0.0
X=0.45, Z=0.02
$M_{WD}=0.55-1.2\ M_\odot$ (0.05 $M_\odot$ step)
UV 1455Å
M_V
log t (day)
$F_{1455Å}\ (10^{-12}$ erg s^{-1} cm^{-2} Å$^{-1})$



fusion and blowing the WD apart. After a Type Ia SN, a BH is left behind. Type Ia SN do not have H in their spectra! This means that all the H has been fused into heavier elements or ejected into space prior to their explotion.

All Type Ia SN have essentially the same magnitude

$$\langle M_B \rangle \simeq \langle M_V \rangle \simeq -19.3 \pm 0.03\,. \tag{48}$$

The spread in this relation is only 0.3 magnitudes. Therefore, since the luminosities are all the same, the flux depends only on the distances. We can use SN Type Ia as "standard candles," in the same way as we did for Cephieds. We will return to this point next semester. Because Type Ia SN are incredibly luminous (as luminous as entire galaxies), we can determine their distances over much of the Universe. Cephieds, although bright for stars, are limited to a smaller volume.

https://www.youtube.com/watch?v=774B8-9B4Ow
https://www.youtube.com/watch?v=DhkWx8-efq0

NS and BH Binaries

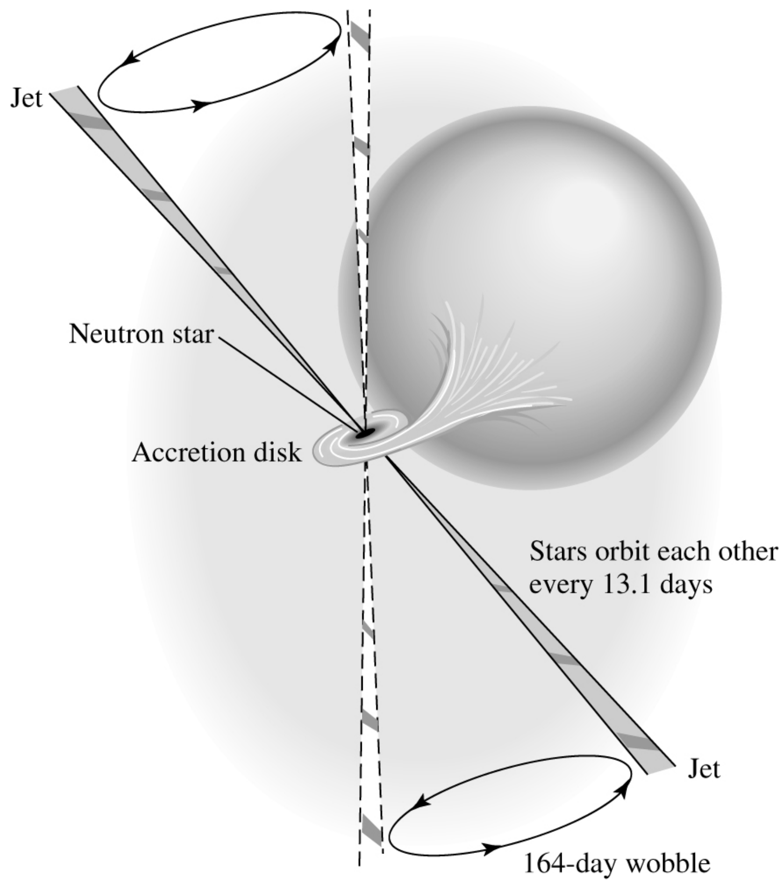
We have mostly been dealing with WD binaries, because their evolution is quite interesting. But what if one of the stars in a binary is massive ($M < 8\ M_\odot$)? In that case, the massive star will go SN quickly. Assuming that explosion doesn't disrupt the binary, we can have a binary with one main sequence star and one NS or BH. Eventually, we can have a NS-NS binary, or a BH-NS binary. A NS-NS binary was actually recently discovered by LIGO! https://www.ligo.caltech.edu/video/ligo20171016v4

The most obvious consequence of a NS or BH binary is that the infalling mass can travel a greater distance. It can therefore get rid of a greater amount of its initial orbital energy, heat up further, and emit at shorter wavelengths. As a result, such systems are strong X-ray emitters. The first source of X-rays outside the Solar system in fact was later found to be an X-ray binary, Sco-1.

As the material funnels onto a NS, it can form a layer on the surface. This surface layer of H and a deeper layer of He can ignite violently, leading to a burst of X-rays. These are called "X-ray bursters."

Millisecond Radio Pulsars

We saw earlier that pulsars are born with periods of about 1 s. Some pulsars are detected with periods down to about 0.001 s, called "millisecond pulsars." Such objects must be



created in a binary system, with accretion providing extra angular momentum to "spin-up" the primary pulsar.

Black Widow Pulsars

A rare situation may arise in a binary with a WD and a pulsar. In this case, the pulsar's intense emission may ablate the secondary, leading to further mass loss. The primary eventually will completely disintegrate the secondary. https://www.youtube.com/watch?v=-SoZ1xvCpMw

Double Neutron Star Binaries

A NS-NS binary will lose energy due to gravitational waves. In this cases, there cannot be mass transfer since the mass is too tightly bound to the individual NSs. The SN nevertheless inspiral due to a loss of energy through gravitational radiation.

The most famous example is the "Hulse-Taylor" binary, which was dscovered by its name-

sakes in 1974. This earled them the 1993 Nobel prize in physics https://en.wikipedia.org/wiki/Hulse%E2%8
The Hulse-Taylor binary is orbiting faster and faster due to this loss of energy. Because the NSs are pulsars, we can measure this decrease in period with exquisite precision. These measurements agree with the predictions of GR, providing one of the strongest pieces of evidence for its validity, as well as providing the first evidence for gravitational waves.

Short-Hard Gamma-Ray Bursts

What happens when two NS eventually collide? Gamma-rays! So-called long-soft gamma rays burst are from core-collapse SN. But short-hard gamma ray bursts are from NS-NS or NS-BH mergers.