ASTR469 Lecture 9: Photometry (parts of Ch. 10)
Chapters 8–9 are not required reading but may be of interest to those who are keen.

1 Photometry

Generically, photometry refers to quantifying the amount of light we receive. The most obvious application is the creation of a 2D image. Remember that all photometric measurements must be taken with a particular filter.

In the past we've spoken about the photons coming out of astronomical objects, and the flux incident on our telescopes. Now it's time to discuss actually measuring the photons. This will involve a practical discussion of the statistics of collecting photons in the presence of noise, which will be applicable to all wavelength regimes. We'll also discuss the most common technology used to measure photons: Charge-Coupled Devices (CCDs).

First, how can we tell if a source is "detected." The signal-to-noise ratio (S/N, sometimes written "SNR") is the important quantity for determining whether a source is detected or not. It is the ratio of the "signal," or total light collected from the source, to the "noise" or the background level (σ). While the signal is relatively easy to understand, noise takes a bit more to comprehend because there are many different sources of noise.

2 Signal

Let's say we have a source flux F_ν in units of $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$ (i.e. $\mathrm{W\,m^{-2}\,Hz_{-1}}$). We integrate for long enough, and the signal strength is just $F_\nu \times t$ (in $\mathrm{J\,m^{-2}\,Hz_{-1}}$), where t is the integration time. So essentially we're counting the total signal collected as the energy per square meter of the detector, at a given frequency. The longer we integrate, the more photons we collect, and the greater the signal.

3 Noise

When we're taking a measurement of anything, we don't get the perfect measurement every time. Instead, there is uncertainty in any measurement, which we call "noise." The noise comes from everything: the telescope and detector, the atmosphere, the background, and even the source itself.

3.1 Gaussian vs. Poisson Statistics

Gaussians:
Before we go on, we have to discuss statistics a bit. Most things you measure vary with a "Gaussian" distribution (also called a "Normal distribution"). The Gaussian distribution is symmetric about the mean value, and occurs when you have an equal chance of measuring more or less of the quantity. This is the traditional bell-shaped curve that applies to many

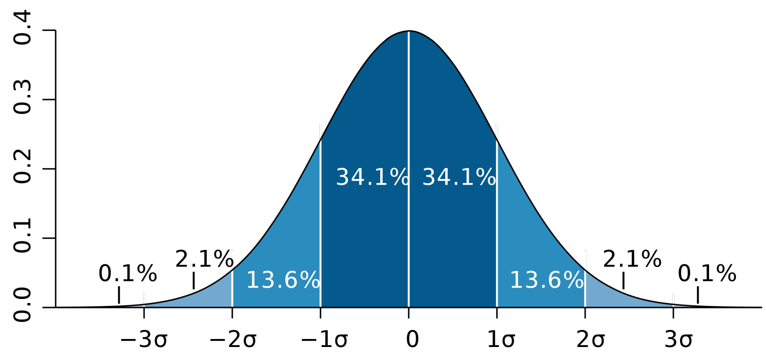

Figure 1: A Gaussian distribution with standard deviation percentiles.

situations, and is given by the probability density of obtaining a measurement value of x:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big(- \frac{(x-\mu)^2}{2\sigma^2}\Big) \tag{1}$$

The most likely signal is μ, which is the mean of the distribution. The breadth of the distribution is characterized by σ. Mathematically, σ is the standard deviation of the measurements, but in practical terms it is also used as the uncertainty in your measurement, which gives the size of the error bar you would use. The "variance" is given by σ^2. The leading constant normalizes the distribution such that the area under the curve is unity.

Gaussian uncertainties are sometimes called "white noise." Imagine measuring some quantity. It is most likely that you measure exactly correctly (at μ), with 68% of your measurements falling within $1\,\sigma$ of the true value, 95.8% within $2\,\sigma$, and 99.8% within $3\,\sigma$.

Poissonians:
The Poisson distribution applies to situations where you're counting discrete events; the event either happens or it doesn't. Events are quantized.

I like to think of the Poisson distribution as resulting from "balls being thrown at a person." Balls are being thrown at me at a constant rate over some period of time. In a given time interval, most of the time I can catch three, but sometimes I only catch two or one, and somewhat more rarely I happen to catch four. Similarly, you might count the number of emails you receive in one day. In astronomy, the discrete events are from photons.

In a Poisson distribution, the uncertainty σ is the square root of the number of events. If there are 100 photons counted, 68% of the time the "true" number of photons will be between $\sqrt{100} = 10$, so between 90 and 110. As N increases, Poisson distributions tend toward Gaussians. Because Gaussian statistics are more convenient, we often assume the noise distribution is Gaussian rather than Poissonian.

4 Noise in astronomy

Let's go through all the sources of noise! We will be using Poissonian statistics.

4.1 Source noise

The source itself of course contributes to the noise in an image:

$$\sigma_{\text{source}} = \sqrt{F_{\text{source}} \times t}\,, \tag{2}$$

where F_{source} is the source flux and t is the time. This is counterintuitive; the noise comes from the fact that we cannot count the source flux with absolute certainty.

4.2 Background/Sky noise

Unresolved sources in the background contribute to the overall noise level:

$$\sigma_{\text{bg}} = \sqrt{F_{\text{bg}} \times t}\,, \tag{3}$$

where F_{bg} is the background flux.

4.3 Adding the Noises and Computing S/N

These noises combine by adding in quadrature. **UNCORRELATED SOURCES OF NOISE NEVER ADD LINEARLY!** If we're targeting an object, then based on the above two noise sources, we get a total noise in our star's measurement of

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{\text{source}} + \sigma^2_{\text{bg}}} \tag{4}$$

(later we will see that our detector introduces some noise, which also adds in quadrature).

Putting it all together, for a star with flux F_{star} and considering only those two noise sources we'd get a S/N ratio of

$$\text{S/N} = \frac{\text{Signal}}{\text{Noise}} = \frac{F_{\text{star}} \times t}{\sqrt{\sigma^2_{star} + \sigma^2_{\text{bg}}}} \tag{5}$$

or

$$\text{S/N} = \frac{F_{\text{star}} \times t}{\sqrt{(F_{\text{star}} + F_{\text{bg}})t}} \tag{6}$$

We frequently use multiples of the noise σ to define the S/N, such that $N \times \sigma = \text{S/N}$. So a 3σ detection has three times the signal compared to the noise. We usually use 3σ as the minimum standard for a detection, whereas 5σ is more secure.

The concept of σ is useful in how it relates to Gaussian statistics. A 1σ detection has a 68.2% chance of being real for a given pixel, 95.6% for 2σ and 99.7% for 3σ. Since there are usually thousands of pixels, 3σ is not usually good enough. A 5σ result has a 99.99994% chance of being real. These probabilities rely on accurate uncertainty estimates.

4.4 How Can I Raise my S/N in an Observation?

There's one cool thing to notice about the previous equation. If you move the t's around, you see that:

$$\text{S/N} = \frac{F_{\text{star}}}{\sqrt{(F_{\text{star}} + F_{\text{bg}})}} \times \frac{t}{\sqrt{t}} = \frac{F_{\text{star}}}{\sqrt{(F_{\text{star}} + F_{\text{bg}})}} \times \sqrt{t} \tag{7}$$

In other words, S/N $\propto \sqrt{t}$ ... whoa!!! **The longer you integrate, the more sensitivity you get**, but it doesn't increase linearly with time (S/N increases more slowly). Important note: this is only true because of the fact that the source's signal becomes brighter with time faster than the noise does. **Integrating more with time will not help if your sky is brighter than your target, or if other non-signal noise sources multiply quickly with time. Sometimes we call this the "bright-source limit."**

5 Charge-coupled devices (CCDs)

CCDs use the photoelectric effect to measure incident light; within a CCD cell, the incoming light strips off an electron and holds it in that cell until it is read out by the user into a computer. CCDs are used from the ultraviolet through the infrared.The web page below illustrates how CCDs work: http://astro.unl.edu/classaction/animations/telescopes/buckets.html

A few important CCD concepts, as applied to astronomy, are:

- We don't detect photons directly! Instead we detect photoelectrons, which are created when photons interact with CCDs.
- To observe in narrow bands, you need external filters; CCDs will otherwise be broadband light detectors. Previously we've spoken about Johnson-Cousins filters in the optical band.
- The number of pixels and pixel size in your CCD can also set your limiting resolution and field of view for a telescope.

CCDs are great for registering light for two particular reasons (among others I'm sure):

- **They have high "Quantum efficiency."** This refers to how many photons are needed to produce one photoelectron, and is also known as "gain." If each photon results in one stored electron, it is 100% efficient. Although not CCDs, our eyes are said to have effective QEs of 3% for rods and 10% for cones. Good CCDs are much better than this, maybe 40% at the most sensitive wavelength. Great CCDs can reach 95% efficiency. The quantum efficiency is wavelength-dependent! Each CCD, depending on how it is manufactured, will have a range of wavelengths where it is most sensitive and therefore has the highest efficiency.
- **CCDs are "linear" detectors.** This means that if you double the number of incident photons, you get twice the stored charge. In other words, the gain is constant irrespective of the incident photon flux. As we know, our eyes are not linear, which led to the magnitude system. However, eventually in a long exposure (or observing something too bright), CCDs will "saturate" (their electron storage capacity in a given pixel can fill), and you will get fewer stored electrons that you expect for a given incident flux.

5.1 Noise specific to CCDs

As previously noted, the noise is composed of every possible source of a signal being registered. CCDs add two in particular...

Read Noise
Read noise comes from the electronics when a CCD is "read out" - when the stored charges are turned into data.

Let r be the read-out noise per pixel (in electrons). The total read out noise is then:

$$\sigma_{\text{read}} = \sqrt{n_{\text{pix}} \times r^2} = \sqrt{n_{\text{pix}}} \times r \tag{8}$$

It may seem strange here to have an r^2 term, and it is, but that is just by convention. We could have just as easily defined it so that the squaring was not necessary.

Dark Current
CCDs build up "dark current" whether they are being exposed to light or not. Recall we discussed Planck spectra for black-body emission; therefore you know that whether or not something is really "bright," just the fact that it's not at 0 K means that it is to some degree emitting photons over broad frequency bands. Dark current is caused by thermally generated electrons that build up in the pixels of all CCDs, causing excess voltage when the CCD is read out.

The rate of dark current accumulation depends on the temperature of the CCD (lower temperatures better) but will eventually completely fill every pixel in a CCD. The rate can be reduced by cooling a CCD.

$$\sigma_{\text{dark}} = n_{\text{pix}} \times dk \times t \tag{9}$$

where dk is the rate for number of electrons per pixel per second.

Putting all noise sources in a CCD observation together,

$$\sigma_{total} = \sqrt{\sigma^2_{\text{source}} + \sigma^2_{\text{bg}} + \sigma^2_{\text{dark}} + \sigma^2_{\text{read}}} \tag{10}$$

And therefore

$$S/N = \frac{F \times t}{\sqrt{\sigma^2_{\text{source}} + \sigma^2_{\text{sky}} + \sigma^2_{\text{dark}} + \sigma^2_{\text{read}}}} \tag{11}$$

or

$$S/N = \frac{F \times t}{\sqrt{(F_{\text{source}} + F_{\text{sky}}) \times t + n_{\text{pix}} \times r^2 + n^2_{\text{pix}} \times dk^2 \times t^2}} \tag{12}$$