# ASTR368 Galaxy Evolution Chapter 26.1

When we observe the Universe, it is obvious that galaxies must have interacted in the past. For example,

- 1. Spiral galaxy disks are not symmetric. They are warped! (> 50% are warped) MW is warped too. Warps can be caused by interactions with other galaxies.
- 2. Hot X-ray emitting gas is in between galaxies caused by stripped gas during interactions
- 3. Some dusty ellipticals exist. Where did the dust come from?
- 4. Polar ring galaxies (similar to above). Ring of gas, dust, young stars around E/S0s
- 5. Ring galaxies
- 6. Shell galaxies (Cartwheel)
- 7. Connected pairs (Antennae)
- 8. Tidal tails
- 9. cD galaxies at center of rich clusters
- 10. Multiple nuclei
- 11. Distant galaxies just look like they are interacting.

Could interactions cause these effects? Yes! Galaxies are relatively closely together (well, lots of space is empty, but galaxies are grouped close together). This is opposite from stars in our own Galaxy. For example, if a star is a tennis ball, the nearest star (Proxima Centauri) is over 1000 miles away, past Wichita Kansas. If the Milky Way is a tennis ball, the LMC is 4 inches away, the SMC is 5 inches away, and Andromeda is just over 5 feet away

### Problem 26.1

What is density of stars in a galaxy? MW has  $\sim 2 \times 10^{11}$  stars in  $\pi (25 \times 10^3)^2 \times 1000 \,\mathrm{pc}^{-3} \simeq 0.1 \,\mathrm{pc}^{-3}$ 

What fractional volume do stars take up in a galaxy?

$$n = 0.1 \text{ pc}^{-3}$$

$$\rho = \langle M \rangle n = 0.5 \ M_{\odot} \times 0.1 \text{ pc}^{-3} = 0.5 \ M_{\odot} \text{ pc}^{-3}$$
radius = 0.5  $R_{\odot}$ 
radius = 0.5  $R_{\odot}$ 
Fractional volume =  $nV_*0.1 \text{ pc}^{-3}(4/3\pi (0.5 \times 7 \times 10^8 \text{m}/3 \times 10^{16} \text{m/pc})^3) \simeq 10^{(-1+0-24)} = 10^{-25}$ 

b) What are the odds of a collision?

$$\begin{split} \mathrm{mfp} &= 1/n\sigma \\ n &= 0.1 \ \mathrm{pc}^{-3} \\ \sigma &= \pi \times (0.5 \times 7 \times 10^8/3 \times 10^{16})^2 = 3 \times 10^{-8} \ \mathrm{pc}^2 \\ \mathrm{mfp} &= 1/(0.1 \times 3 \times 10^{-8}) = 3 \times 10^8 \ \mathrm{pc} = 3 \times 10^5 \ \mathrm{kpc} \\ \mathrm{Odds} \ \mathrm{of} \ \mathrm{collision} &= 1 \ \mathrm{kpc}/3 \times 10^5 \ \mathrm{kpc} \simeq 0.001\% \end{split}$$

So no collisions! Some basics of interactions:

- 1. Stars basically do not interact. They are solid bodies (cue balls) and must pass near each other to feel any interaction. Gravity is  $r^{-2}$ , so they have to get pretty close to interact individually.
- 2. Gas is collisional! If you slam two gas clouds together, there will be friction from the gas particles.

### Stellar Encounters

Let's talk about the stars. A "strong" encounter is one in which the change in stellar kinetic energy is approximately equal to the initial kinetic energy.

At t = 0,  $K = 1/2m_1v^2$  and U = 0. Some time later, when stars separated by distance b (the "impact parameter"),  $U = Gm_1m_2/b^2$ . By definition of a strong encounter,  $U = K_i$  and  $Gm_1m^2/b^2 \simeq 1/2m_1v^2$  So  $b = 2Gm_1/v^2$ .

How likely is this? Let's take a cylinder of radius b. Average interaction distance = mean free path =

$$d \simeq 1/(n\sigma) \tag{1}$$

where  $\sigma$  is the cross section. This is a great relationship to memorize! So number of interactions per second  $\simeq v/d$  and time between interactions

$$t_{\rm strong} \simeq \frac{d}{v} = \frac{1}{v n \pi b^2}.$$
 (2)

The number density of stars in Solar neighborhood is  $\sim 0.1 \,\mathrm{pc}^{-3}$ , so  $t_{\mathrm{strong}} = 10^{15}$  years = never!

Now, in dense environments like the center of a globular cluster,  $n \simeq 10^5 \,\mathrm{pc}^{-3}$ ,  $t_{\mathrm{strong}} \simeq 10^9 \,\mathrm{years} = \mathrm{almost}$  never.

So, if you collided two galaxies together, the odds are that no two stars would have a strong interaction! This of course depends on the velocity of the collision, and whether the densest parts of the galaxies collided.

### **Dynamical Friction**

OK, so strong interactions are rare, but "dynamical friction" is important. If mass M moves through sea of stars with similar masses m, all other stars are slightly attracted to the moving mass. The displacement of each attracted mass may not be much, but together they form a "wake" behind the moving mass. This wake in turn attracts the moving mass in the direction opposite its motion, slowing it down.

The net result is that the moving mass is slowed, even though there may not be any strong interactions. This is called dynamical friction.

In general, a simplified equation for the force from dynamical friction has the form

$$F_{\rm dyn} \approx C \frac{G^2 M^2 \rho}{v^2} \tag{3}$$

where the C is not a constant, but instead depends on how v compares to the velocity dispersion of the surrounding matter. C has a representative value for all systems = 23 for LMC, 76 for globulars, 160 for ellipticals [must obviously just be representative].

We didn't derive this equation, but we can understand the form.

- We know that the interaction must depend on the density  $\rho$ .
- The mass is squared and one power comes from the gravitational interaction from the "wake" as we would expect.
- The other power comes from actually producing the wake.
- The velocity squared term comes from the inverse square law for gravity.

If the mass moves twice as fast, it will be twice as far away when the wake forms. Therefore the inverse square law requires the  $v^2$  term.

### Interaction Timescales

But how long do these interactions take? The book goes into a nice derivation for globular clusters. Dynamical friction on a globular cluster will cause it to eventually spiral in to its host galaxy. For a  $1/r^2$  density distribution of something of mass M, using above dynamical friction equation, the time is

$$t = \frac{2\pi v_{\max} r_i^2}{CGM} \tag{4}$$

where  $r_i$  is initial radius. If  $v = 250 \text{ km s}^{-1}$ , r = 4 kpc, C = 100,  $M = 5 \times 10^6 M_{\odot}$ , we get  $t = 6 \times 250 \times 10^3 \times (4 \times 3e \times 10^{19})^2 / (100 \times 7 \times 10^{-11} \times 1 \times 10^6 \times 2 \times 10^{30})$ , which reduces to  $t = 10^{(6+40-2+12-37)} = 10^{17} \text{ s} = 10^{10}$  years.

This works out to periods of GYrs for all manner of currently interacting systems. t goes down as  $v_{\text{max}}$  decreases, r decreases, and M increases.

We can turn this around so

$$r_{\rm max} = \frac{t_{\rm max} CGM^{0.5}}{2\pi v_{\rm max}}.$$
(5)

This gives us the radius within which a cluster can be swallowed by the host galaxy, for a given mass, velocity, and age of the galaxy.

### Galaxy Formation - Chapter 26.2

Galaxies exist! They must have formed somehow - how? We have some lines of evidence that all formation scenarios must account for:

- Halo stars have strange orbits some orbit opposite of galactic rotation
- Halo stars are metal poor.
- Galactic disk more metal rich.
- Globular clusters have a wide spread in ages, from 3 Gyr to 14 Gyr.
- Two types of globular cluster: metal poor and metal rich.

• Dark matter not nearly as centrally concentrated as light.

[Review metallicity: first stars very metal poor, then exploded as SN, enriched ISM, next generation of stars formed with higher metallicity]

There are two models of galaxy formation. This lecture will spend a lot of time on the first because it is conceptually the easiest. But it's almost certainly wrong.

The first is known as "top-down" (Eggen, Lynden-Bell, & Sandage, 1962). "Top-down" here refers to the idea that the galaxy formed from one large entity. The opposite is "bottom-up" where the galaxy forms from many smaller groups of stars+gas.

In the top-down scenario, the galaxy formed from a large proto-galactic cloud that collapsed. This is analogous to the formation of a star (and the Solar System).

How fast was this collapse? If free fall,

$$t_{\rm ff} = \left(\frac{3\pi}{32} \frac{1}{G\rho}\right)^{0.5} \simeq \left(\frac{1}{G\rho}\right)^{0.5} \tag{6}$$

If  $\rho = 3M/(4\pi R^3) = 10^{-25} \text{ g cm}^{-3}$  for  $M = 6 \times 10^{11} M_{\odot}$  and R = 100 kpc. This leads to  $t_{\text{ff}} = 7 \times 10^8$  years. Lots of assumptions! Characteristic density needed, maybe radius not right, etc etc.

Why may the free fall time not be correct? It assumes that the energy can radiate away freely, which may be the largest assumption.

[Another important timescale here is the crossing time. This one is straightforward. Given the size of a cluster and the typical velocity of galaxies in it, how long does it take a galaxy to move from one side to the other?]

As a gas cloud collapses, the transfer of potential energy into kinetic will naturally heat up the cloud. This is a problem in star formation as well. Let's try to calculate a "cooling time" that we can compare to the free fall time. If the cooling time is larger than the free fall time, then the proto-galactic cloud cannot collapse.

The Virial Theorem is useful here, because the extra "K" is basically just radiated energy and we can assume that the proto-galaxy is gravitationally bound. From the usual derivation:

$$-2\langle K\rangle + \langle U\rangle = 0 \tag{7}$$

$$-2(1/2N\mu m_H)\langle v^2 \rangle = -3/5 \frac{GM^2}{R} \,, \tag{8}$$

where  $\mu$  is the mean molecular weight and N is the number of particles.

$$1/2N\mu m_H = M \tag{9}$$

$$= \langle v^2 \rangle^{0.5},\tag{10}$$

$$\sigma = (3/5\frac{GM}{R})^{0.5} \tag{11}$$

[we had this before, minus the factor of 3 that comes from what is actually observed]

 $\sigma$ 

$$1/2\mu m_H \sigma^2 = 3/2k T_{\text{virial}} \tag{12}$$

$$T_{\rm virial} = \frac{\mu m_H \sigma^2}{3k} \tag{13}$$

So now we have a characteristic temperature. How quickly can energy be radiated? Need: [volume cooling rate in energy \* volume / time] known as (cooling function  $\Lambda$ ) This cooling rate \* density \* time is just the total energy, which is the same as the kinetic energy.

$$\Lambda \times \text{density} \times t_{\text{cool}} = 3/2NkT_{\text{virial}} \tag{14}$$

 $\operatorname{So}$ 

$$t_{\rm cool} = 3/2 \frac{kT_{\rm virial}}{n\Lambda} \tag{15}$$

For the Milky Way, using the cooling function value presented in your book,  $t_{\rm cool} \simeq 10^7$  years.

The cooling function depends on the characteristic temperature of the gas, because different emission mechanisms are possible in different regimes (show slide)

- Why is it more efficient cooling at high metallicity? - Why is it more efficient cooling after  $10^4$  K?

A collapsing cloud can radiate away the energy fast enough to collapse at the free fall speed. More massive though,  $T_{\text{virial}}$  will go up (deeper potential well leads to higher velocity dispersion), and  $t_{\text{cool}}$  will also go up - cannot radiate away energy fast enough.

Interestingly, this accounts for dark matter distribution! Dark matter cannot radiate energy, and so maintains its larger distribution.

While interesting, the top-down hypothesis is almost certainly wrong!

- Halo stars should not be on retrograde orbits
- Halo stars not necessarily metal poor (unless collapse slower here)
- Galactic disk not necessarily more metal rich (unless collapse faster here)
- Globular clusters should not have a large spread in ages (collapse more uniform everywhere)
- Ditto for spread in metallicities.
- Dark matter thought to be clumpy, not smooth as would be expected.

### **G-dwarf** Problem

There should be more low metallicity stars! The first generation of stars formed with essentially zero metallicity. These stars have very long lifetimes, and should still be around. We do not find enough F and G stars that have low metallicities. Three solutions:

- 1. Enhancement was rapid
- 2. Unenriched material constantly falling onto disk
- 3. More massive stars produced early on, more F and G type stars today.

Second model called "bottom up" and this is what is favored today. Small  $10^6$  to  $10^8 M_{\odot}$  "proto galactic fragments" merge into a large potential well. These fragments initially form stars individually, like globulars. Collapse more rapid at center of proto-galaxy (stars formed there first, oldest stars today).

- 1. Explains retrograde halo stars (individual halo stars from disrupted fragments
- 2. Does explain metallicity (high in center, low in halo)

- 3. Range of GC ages and metallicities
- 4. DM clumpy

Also, globular clusters all have  $10^5$  to  $10^6 M_{\odot}$ . Force of dynamical friction goes as M2 so largest ones destroyed. Smallest ones lack gravity to hold them together. Argument works if lots of collisions between protogalactic filaments Collisions more frequent at center of galaxy, which makes stars in bulge earlier (older population).

[show simulation]

# ASTR368 Active Galaxies Chapter 28

#### 28.1

Edward Faith, when studying "spiral nebulae" in 1908 noticed that some of them had emission lines. This is strange. Most galaxies have absorption lines, due to the fact that their emission comes from stars and stars have absorption lines. Carl Seyfert reported in 1943 that these emission lines are from bright nuclear regions that exist in a small percentage of galaxies. Due to the electronic transitions involved, these lines must be from hot gas.

An Active Galactic Nucleus (AGN) is a compact central region of a galaxy with higher than normal luminosity. We call the host galaxy an "Active galaxy".

- Excess emission can span entire EM spectrum, from radio to infrared, optical, X-ray, gamma ray
- The excess emission is related to the black hole at the center

Why are black holes bright? Black holes are obviously regions that are so dense that light cannot escape. How big are they? Quick (but flawed) derivation from last semester:

$$v_{\rm esc} = c \tag{16}$$

$$1/2mv_{\rm esc}^2 = GMm/r \tag{17}$$

$$v_{\rm esc} = (2GM/r)^{0.5} = c \tag{18}$$

$$r = 2GM/c^2 = 3 \,\mathrm{km} \,\mathrm{per} \,\mathrm{Solar} \,\mathrm{mass} \tag{19}$$

(how should we do this derivation 100% correctly? Use GR!) OK back to why would they be bright.... Accretion!

Disks usually rotate such that each fluid element is moving almost (but not exactly!) in a circular orbit. If there were no interactions between fluid elements, what would the angular velocity be as a function of radius?  $\Omega \propto R^{-3/2}$  (Keplarian). But in a fluid this sets up a shearing flow, or a friction, or a force.

Given that the outer parts rotate more slowly, in which direction will the force be and what will be the effect on the angular momentum and on the movement of mass? Inner tries to speed up outer, giving it a higher velocity. This increases the angular momentum of the outer, decreases the angular momentum of the inner, so net result is that angular momentum is transferred outwards and mass flows inwards (some subtleties, of course).

So, gas moving towards a massive object has a tendency to circularize, form a disk, and spread inward and outward. This is an "accretion disk". If the massive object has a surface, then often the matter spirals in

until it hits the surface or interacts with the object's magnetic field, whichever comes first. But a black hole has neither a surface nor a magnetic field.

The energy efficiency for black hole accretion can in principle be the highest in astrophysics. Unlike stars, for black holes all the emitted energy must come from the accretion disk. We will therefore take a closer look at accretion disks.

# Temperature and frequency distribution of a thin disk

Suppose that as each fluid element moves inward it releases its energy locally, and that its energy is all gravitational. How much energy would an element of mass m release in going from a circular orbit at radius r + dr to one at radius r?

Gravitational potential energy is

$$E_g = -\frac{GMm}{2r}\,,\tag{20}$$

so the energy released is

$$E = \frac{GMmdr}{2r^2}.$$
(21)

The Virial Theorem tells us that half the energy is released, and half goes into the potential, so

$$dE_g \simeq \frac{GMmdr}{r^2} \,. \tag{22}$$

That means that the luminosity of this annulus, for an accretion rate  $\dot{M}$ , is

$$dL = \frac{dE}{dt} \simeq \frac{Gm\dot{M}dr}{r^2}.$$
(23)

What is the temperature, assuming the annulus radiates its energy as a blackbody? For a blackbody,  $L = \sigma AT^4$ . The area of the annulus is  $2\pi r dr$ , and therefore

$$L \simeq \frac{M\dot{M}dr}{r^2} = 2\pi\sigma T^4 r dr \tag{24}$$

 $\mathbf{SO}$ 

$$T^4 \simeq \frac{MM}{r^{-3}} \tag{25}$$

or

$$T \simeq \left(\frac{M\dot{M}}{r^3}\right)^{1/4}.$$
 (26)

Therefore, the temperature increases as the fluid moves in. Another point is that from this equation we can see general scalings with the mass M of a central black hole.

This temperature dependence leads to X-ray emission close to the black hole, moving out toward infrared light further away.

AGN have bright emission over a large range of wavelenghts. Big Blue Bump and IR emission probably from accretion disk. In the radio regime, synchrotron dominates.

# Types of active galaxy

It is convenient to divide AGN into two classes, conventionally called radio-quiet and radio-loud. Radio-loud objects have emission contributions from both the jet(s) and the lobes that the jets inflate. These emission contributions dominate the luminosity of the AGN at radio wavelengths and possibly at some or all other wavelengths. Radio-quiet objects are simpler since jet and any jet-related emission can be neglected at all wavelengths. AGN terminology is often confusing, since the distinctions between different types of AGN sometimes reflect historical differences in how the objects were discovered or initially classified, rather than real physical differences.

### Radio-quiet AGN

Low-ionization nuclear emission-line regions (LINERs). As the name suggests, these systems show only weak nuclear emission-line regions, and no other signatures of AGN emission. It is debatable whether all such systems are true AGN (powered by accretion on to a supermassive black hole). If they are, they constitute the lowest-luminosity class of radio-quiet AGN. Some may be radio-quiet analogues of the low-excitation radio galaxies (see below).

#### Seyfert galaxies

Seyferts were the earliest distinct class of AGN to be identified. They show optical nuclear continuum emission, narrow and occasionally broad emission lines, occasionally strong nuclear X-ray emission and sometimes a weak small-scale radio jet.

Originally they were divided into two types known as Seyfert 1 and 2: Seyfert 1s show strong, broad emission lines while Seyfert 2s do not, and Seyfert 1s are more likely to show strong low-energy X-ray emission.

### Radio-quiet quasars/QSOs

These are essentially more luminous versions of Seyfert 1s: the distinction is arbitrary and is usually expressed in terms of a limiting optical magnitude.

Quasars were originally "quasi-stellar" in optical images as they had optical luminosities that were greater than that of their host galaxy. Star-like objects whose emission lines initially didn't match with any known elements. Later realized that the emission lines were that of hydrogen redshifted an amazing amount, indicating that they were really far away (see Hubble's Law later)

They always show strong optical continuum emission, X-ray continuum emission, and broad and narrow optical emission lines. Some astronomers use the term QSO (Quasi-Stellar Object) for this class of AGN, reserving 'quasar' for radio-loud objects, while others talk about radio-quiet and radio-loud quasars. The host galaxies of quasars can be spirals, irregulars or ellipticals. There is a correlation between the quasar's luminosity and the mass of its host galaxy, in that the most luminous quasars inhabit the most massive galaxies (ellipticals).

### Radio-loud AGN

Radio-loud quasars behave exactly like radio-quiet quasars with the addition of emission from a jet. Thus they show strong optical continuum emission, broad and narrow emission lines, and strong X-ray emission, together with nuclear and often extended radio emission.

### Blazars (BL Lac objects)

classes are distinguished by rapidly variable, polarized optical, radio and X-ray emission. BL Lac objects show no optical emission lines, broad or narrow, so that their redshifts can only be determined from features in the spectra of their host galaxies. The emission-line features may be intrinsically absent or simply swamped

by the additional variable component. In the latter case, emission lines may become visible when the variable component is at a low level.

#### **Radio galaxies**

These objects show nuclear and extended radio emission. Their other AGN properties are heterogeneous.

The central black hole of AGN frequently powers radio jets. The jets form as a result of the accretion, to get rid of excess angular momentum. The jets themselves consist of plasma, but their strong magnetic fields in the jets leads to synchrotron emission.

At least half of radio galaxies have jets

# A unified model of AGN 28.2

It is clear that AGN are powered by central supermassive black holes. Given that they are all caused by the same physical phenomenon, it would seem strange if all the various types were really all distinct. It is hypothesized that there is actually a unified model of AGN. Some evidence for this:

- if you look in polarized light, which in general has a much lower optical depth, toward Seyfert 2's (broad lines) you may see Seyfert 1's (only narrow lines). It's as if there were Seyfert 1's lying within Seyfert 2's, bu their emission was obscured.
- The H $\alpha$  vs continuum plot containing various AGN types is a straight line (Figure 28.21). Both of these are related to the ionizing photon rates. The fact that they are strongly correlated shows that there is a common origin to the observed properties of the various types.

AGN are powered by the accretion disk surrounding supermassive black holes. The reduction of gravitational potential energy as mass moves in toward the black hole powers the luminosity. Somehow this must result in the wide variety of observational signatures seen in AGN. Things to notice:

- Why would the narrow line region be further out?
- Why would some galaxies be radio loud and others radio quiet?
- Seyfert 1s have only narrow lines because the broad line region is obscurred
- Seyfert 2s have broad lines
- Radio quiet AGN the jet itself can be obscured
- Blazars, which have rapid time-variability, we see directly down the jet.

A few other points about AGN: How big is the black hole?

Assume we have this geometry, with the emitting region represented by radius R. Light from the near side of this region has a path length of  $l_1$  whereas light from the edge of the region has path length  $l_2$ . We can't see the back of the sphere because it is optically thick. From the geometry:

$$l_2 = \frac{l_1 + R}{\cos \theta} \simeq l_1 + R \tag{27}$$

since  $\cos \theta \simeq 1$ . Light from the near side must travel an additional distance  $l_2 - l_1 \simeq R$ . Therefore the brightness is smeared out over  $\Delta t = R/c$ . The quantity R/c is the light crossing time.

We know from observations that AGN can vary on timescales of ~ 1 hour. Ignoring relativistic effects, this gives a characteristic size of  $3600 \text{ s} \times 3 \times 10^8 \text{ m/s} = 1 \times 10^{12} \text{ m} = 7 \text{ AU}$ . That's pretty small!

For a typical black hole,  $R_s = 3 \text{ km}$  per Solar mass, so a  $10^6$  Solar mass black hole would be  $3 \times 10^6 \text{ km} = 0.02 \text{ AU}$ . Even smaller!

## Superluminal motion

Remember how we can measure the motion of knots of plasma in a radio jet. Well, it turns out that they are apparently moving faster than the speed of light. Uh oh.

Assume a knot of gas emits a photon at t=0 when the distance to earth is d, and then some time later at  $t = t_e$  emits a second photon. At the time the second photon is emitted, the distance to earth is

$$d_2 = d - vt_e \cos\phi. \tag{28}$$

The first photon reaches Earth at time

$$t_1 = d/c \tag{29}$$

and the second at

$$t_2 = t_e + d_2/c = (d - vt_e \cos \phi)/c$$
(30)

The time between when these are received is

$$\Delta t = t_2 - t_1 = t_e (1 - v/c \cos \phi) \tag{31}$$

Notice that for most values of  $\phi$  this is a fraction of  $t_e$ ! How could that be? We are witnessing the transverse velocity of the knot. Using  $v_{app}$  for the transverse velocity,

$$v_{\rm app} = \frac{vt_e \sin \phi}{\Delta t} = \frac{v \sin \phi}{1 - (v/c) \cos \phi} \tag{32}$$

And therefore

$$\frac{v}{c} = \frac{(v_{\rm app}/c)}{(\sin\phi + (v_{\rm app}/c)\cos\phi)}$$
(33)

or if  $\beta_{\text{app}} = v_{\text{app}}/c$  and  $\beta = v/c$ :

$$\beta = \frac{\beta_{\rm app}}{\sin\phi + \beta_{\rm app}\cos\phi} \tag{34}$$

or

$$\beta_{\rm app} = \frac{\beta \sin \phi}{1 - \beta \cos \phi} \tag{35}$$

If we take  $\beta = 0.99$  and  $\phi = 10^{\circ}$ , we get  $\beta_{app} = 6.9$ . This is "superluminal motion."

As  $\beta$  decreases, or if  $\phi$  increases, this effect is minimized (show figure).

When is the superluminal motion at a maximum? I'll leave it as an exercise to show

$$\phi_{\max} = \cos^{-1}\beta \tag{36}$$

and

$$\beta_{\rm app,max} = \frac{\beta}{(1+\beta^2)^{0.5}} \tag{37}$$

and

$$\beta_{\min} = 0.707 \tag{38}$$

Motion of jet need not be highly relativistic to get superluminal motion.