ASTR 368
HW #5

1) a) $v = H_0 d = 7{,}000$ km/s

$H_0 = 67$ km/s /Mpc

$\Rightarrow d = v/H_0 = 7{,}000/67 = 105$ Mpc

b) $z = \frac{v}{c} = 0.023$ non-rel.

$z = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} - 1 = \left(\frac{1+v/c}{1-v/c}\right)^{1/2} - 1 = \left(\frac{1+0.023}{1-0.023}\right)^{1/2} - 1$

$= 0.024$ rel.

c) $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \Delta\lambda = 656\text{nm} \cdot 0.023 = 15.1$ nm

(or 15.7 nm for rel.)

d) $m_V - M_V = 5\log d - 5$

$m_V = -19.3 + 5\log(105\times10^6) - 5 = 15.8$

e) $M_V = -3.53 \log P_d - 2.13 + 2.13(B-V)$

for simplicity, assume $B-V=0$

$M_V = -3.53 \log 14 - 2.13 = -6.17$

$m_V = m_{V,SN} + 13.1 = 28.9$

f) $d = \frac{1}{p} \Rightarrow p = \frac{1}{d} = \frac{1}{105 \times 10^6} = 9.5 \times 10^{-9}$ "

Hubble resolution is $1/10$ "

c) $M = \frac{5 \cdot 3\,\text{Mpc} \cdot (1000\ \text{km/s})^2}{G}$

$= 6.9 \times 10^{45}\ \text{kg} = 3.5 \times 10^{15}\ M_\odot$

d) $t_{cross} \sim \frac{R}{\sigma} = \frac{3\ \text{Mpc}}{1000\ \text{km/s}} = 9.26 \times 10^{16}\ \text{s} = 2.93 \times 10^{9}\ \text{yr}$

e) Since $t_H \sim 14 \times 10^9$ yr, $t_H \gg t_{cross}$ and it should be virialized.

2) a) $dU = -G \frac{M_{int}\, dm}{r}$

$$U = -G \int_0^r \frac{M_{int}}{r}\, dm$$

$dm = 4\pi r^2 \rho\, dr \qquad M_{int} = 4/3 \pi r^3 \rho$

If $\rho \neq \rho(r)$, $\quad U = -4\pi G \rho \cdot 4/3\pi \rho \int_0^r r^4\, dr$

$$= -\frac{16\pi^2}{3} G\rho^2 \cdot \frac{1}{5} r^5$$

$$= \frac{-16\pi^2}{15} G\rho^2 r^5$$

$M = 4/3 \pi r^3 \rho \Rightarrow \rho = \frac{3M}{4\pi R^3}$

$$U = \frac{-\cancel{16}\cancel{\pi^2}}{\cancel{15}\, 5} G \cdot \frac{\cancel{9}\, 3 M^2}{\cancel{16\pi^2} R^6} R^5 = \frac{-3}{5} \frac{GM^2}{R}$$

b) $2K + U = 0$

$K = 1/2 M\sigma^2 \qquad U = -\frac{3}{5}\frac{GM^2}{R}$

$$\cancel{M}\sigma^2 = \frac{3GM^{\cancel{2}}}{5R}$$

$$M = \frac{5R\sigma^2}{3G}$$

since $\sigma_r^2 = 1/3\, \sigma^2$, $\qquad M = \frac{5R\sigma_r^2}{G}$