Stellar - HW2
August 29, 2024, Due September 5, 2025
2 pt for each question part

1) A useful (albeit not terribly realistic) model for a homogeneous composition star may be obtained by assuming that the density is a linear function of the radius:

$$\rho(r) = \rho_c[1 - r/R], \tag{1}$$

where ρ_c is the central density and R is the total radius where we can assume $P(R) = T(R) = 0$ apply. There is a lot of algebra in the following steps but the integrals are easy to do analytically.

a) Find an expression for the central density in terms of R and M. (Use the mass equation!)

b) Use the equation of hydrostatic equilibrium and the boundary conditions to find the pressure as a function of radius. Your answer will be of the form $P(r) = P_c \times$ (polynomial in r/R), where P_c is the central pressure. What is P_c in terms of M and R? (It should be proportional to GM^2/R^4). Express P_c numerically with M and R in solar units.

c) In this model, what is the central temperature, T_c? Assume an ideal gas. Compare this result for that we obtained with the constant density model and try to explain the difference.

d) Verify that the virial theorem is satisfied and write down an explicit expression for the gravitational potential energy Ω (i.e. what is α?)

e) (Grad students only) Please repeat 1a - 1d for the density function

$$\rho(r) = \rho_c[1 - r/R]^2. \tag{2}$$

2) Assume that the density at the Solar photosphere is 2.0×10^{-7} g/cm^3 and the opacity in the photosphere is $0.3\,\text{cm}^2$/g. Estimate the depth at which light is emitted from the Sun when viewed face-on (assume it is emitted at $\tau = 2/3$).